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3044 WEST 106TH STREET • CLEVELAND 11, OHIO

Model 160

PROBABILITY DENSITY ANALYZER

Instruction Manual

BOLT BERANEK AND NEWMAN INC

CONSULTING • DEVELOPMENT • RESEARCH

REPORT NO. 895

PROBABILITY DENSITY ANALYZER

SUMMARY REPORT

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SUMMARY REPORT

PROBABILITY DENSITY ANALYZER

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PROBABILITY DENSITY ANALYZER ... SUMMARY REPORT

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	1
	A. Historical Summary	1
	B. The Scope of This Report	3
II.	PROBABILITY DENSITY	5
III.	THEORY OF MEASUREMENT OF PROBABILITY DENSITY	10
IV.	PROBABILITY DENSITY ANALYZER - GENERAL THEORY	14
	A. Logical Analysis - Operations	14
	B. Functional Analysis - Waveforms	15
	C. Functional Analysis - Block Diagram	20
V.	USE OF THE PROBABILITY DENSITY ANALYZER	23
VI.	THE B + K MODEL 160 PROBABILITY DENSITY ANALYZER -- DETAILED DISCUSSION	37
	A. Operational Procedures	37
	B. Detailed Theory	64
	C. Alignment	72
	D. Performance Data	83
	E. Specifications	92

TABLE OF CONTENTS CONT.

Page

APPENDICES:

A. UNCERTAINTY IN ESTIMATES OF THE PROBABILITY DENSITY A-1

B. EXAMPLES OF COMMON PROBABILITY DENSITIES B-1

C. DEVIATION COEFFICIENTS C-1

D. METHODS OF CALIBRATION D-1

LIST OF ILLUSTRATIONS iii

LIST OF TABLES vi

REFERENCES . . . . . a

INDEX c

ILLUSTRATIONS

Figure		Page
1.	Graphical Illustration of the Meaning of the Estimator	11
2.	Signal Processing Requirements to Measure Probability Density	16
3.	Sequence of Waveforms Representative of the Signal Processing as Indicated in Fig. 2	18
4.	Functional Block Diagram of B + K PDA Model 160	21
5.	The Sensitivity of Probability Density Analyzer to Certain Types of Waveform Distortions -- Example	24
6.	Probability Distribution of Strain Maxima for Several Excitation Levels of Single Degree of Freedom System (after Smith and Lambert)	26
7.	Probability Density Curves for the Envelope of the Response of a Tuned Circuit to Three Superposed Swept Frequencies (after Arguimbau, BBN Report No. 685)	27
8.	Theoretical Distribution of Instantaneous Values Obtained from the Combination of n Cosine Waves (after Slack)	28
9.	Probability Densities Representing the Sum of Two Independent Cosine Waves for Various Energy Ratios (after Fox and Starr)	30

ILLUSTRATIONS CONT.

Figure		Page
10.	Probability Density of the Sum of Independent Square and Sine Waves for Equal Energy Ratios (after Fox and Starr)	31
11.	Probability Density of the Sum of Independent Square, Cosine and Triangular Waves for Equal Energy (after Fox and Starr)	32
12.	Probability Density of Pressures at Various Distances from Pulse Sources in "Hard Room" (after Lyon)	33
13.	Gas Tube Noise Showing Non-Gaussian Characteristic	35
14.	Silicon Diode Noise Showing Non-Gaussian Characteristics	36
15.	Circuit Diagram -- Probability Density Analyzer, Model B	65
16.	Typical Waveforms in the Arguimbau Circuit	68
17.	Circuit Diagram -- Power Supply	73
18.	Waveforms (see text for input and circuit conditions). Location: pin 2 V 14	75
19.	Waveforms (see text for input and circuit conditions). Location: Test point D	76
20.	Waveforms (see text for input and circuit conditions). Location: Test point E	78
21.	Waveforms (see text for input and circuit conditions). Location: pin V 9	79

ILLUSTRATIONS CONT.

Figure		Page
22.	Waveforms (see text for input and circuit conditions). Location: Test point G	81
23.	Waveforms (see text for input and circuit conditions). Location: Test point H	82
24.	Probability Density of 100 cps Sine Wave, Measured by Model 160B	84
25.	Probability Density of 1 kc Sine Wave, Measured by Model 160B	85
26.	Probability Density of 10 kc Sine Wave, Measured by Model 160B	86
27.	Probability Density of Gas Tube Noise Generator "Speech" Spectrum Shape Measured by Model 160B	87
28.	Probability Density of 20 kc Sine Wave, Measured by Model 160B	88
29.	Probability Density of 30 kc Sine Wave, Measured by Model 160B	89
30.	Probability Density of Gas Tube Noise Generator Output > 20 kc Bandwidth	90

TABLES

Table		Page
A-I	Confidence Levels to Determine That c% of the Measurements are in Error Within $p\% = K\sigma_n\%$	A-4

I. INTRODUCTION

The B + K Model 160 Probability Density Analyzer (PDA) is the result of a development program conducted by Bolt Beranek and Newman (BBN) for B + K Instruments of Cleveland. It is the purpose of this report to summarize that work, to describe the instrument, to outline methods of application, and to describe some researches which have already made use of the instrument. It is emphasized that this report is based not only on the prototype instrument developed at BBN but is based also on the production instrument, the Model 160, several of which are operational in government and commercial laboratories.

A. Historical Summary

Although the Model 160 has resulted from a specific development program, it is actually the culmination of several years of work in statistical analysis instrumentation for sound and vibration data. A characteristic feature of the work in vibration and sound in the last decade has been the wide spread introduction of the concept of randomly fluctuating data. The pioneering role of BBN personnel is best exemplified by the (now classic) chapter 10 of L. L. Beranek's book, Acoustic Measurements, published in 1949. By 1955 primitive instrumentation had been assembled as was reported by Smith.[1/] There followed many similar attempts, the continued work by Korn and co-workers at the University of University of Arizona being exemplary.

By 1959 it was clear that probability density analysis was essential in many engineering applications, but it was equally clear that the existing approaches were inadequate. The most obvious inadequacy was related to the manner in which the

probability density was measured. It is instrumentally trivial to measure the probability distribution, that is, the probability of exceeding a given amplitude.* This is accomplished by merely establishing a threshold trigger and electronically integrating the resulting pulses. The early workers extrapolated from this concept to measure probability density; they established two levels and took the difference to obtain probability density (see Fig. 1 and associated text). Such was the approach used by Fox at BBN in the statistical analyzer developed for the Army Ballistic Missile Agency. Most of the early work used available analog computer components but in 1959 the use of optico-electronic means was reported in the work of Lien[2] and Broch.[3]

At this juncture B + K realized that to maintain its position in the vanguard of acoustical and vibration instrumentation it was obligatory to supply the industry with a high quality commercial Probability Density Analyzer. The instrument should take advantage of the research to date but should push beyond the state of the art where possible. BBN was contracted to carry out such a program.

BBN was in a unique position with respect to this problem having available both experience with random excitation and sound and sophisticated instrumentation experience as well as a specific capability in the general problems of random signal processing. It was proposed, therefore, in the first phases of the work to examine the most suitable way to obtain probability analysis commensurate with the requirements of random vibration and sound

* The probability distribution can be obtained from the probability density. See page 63 et seq.

data. One of the results of this analysis was the establishment of a theory of the measurement of probability density. This is outlined in section III of this report. The requirements established by an assessment of the types of signals encountered indicated that the existing techniques of obtaining the necessary measurements were not adequate. The result of further research was the invention of a unique method of implementing the instrumentation for obtaining the probability density. This was the work of L. B. Arguimbau.[4]

BBN constructed four prototypes of the PDA in addition to the breadboard. These four units provided valuable field experience. The critical evaluation performed by the U. S. Naval Air Station at Pt. Mugu, and at BBN indicated some desirable modifications. These changes were related specifically to performance when signals at the limit of the specified ranges were used. To insure that the performance would never be marginal, improvements were incorporated to extend the performance beyond that which was to be specified for the commercial instrument.

The description of the prototype PDA was covered in a tentative instruction manual prepared by BBN. Material for updating the manual to cover the modified instrument is presented in this report.

B. The Scope of this Report

The technical material in this report refers to the final hardware. In section II following, background on the concept of probability density is presented to facilitate understanding the subsequent discussion. Section III presents the theory of measure-

ment and follows essentially a presentation made at the May 1961 ASA meeting by H. Fox.[5] In section IV the general theory of the PDA is presented. The specific implementation of the instrumentation concept is left to the detailed material of section VI.

To emphasize the function of a probability density analyzer section V presents some measurements made with the instrument. Some programs which at present are using, or which will use, or would be enhanced by tne use of probability density measurement are also discussed.

Section VI discusses the Model 160. A variety of operational procedures are presented. The detailed theory of the probability density analyzer is discussed. The technique of alignment and calibration of the instrument is presented, and the performance data and specifications conclude section VI. Some special techniques and problems are discussed in the appendices.

II. PROBABILITY DENSITY

Since the work on the Probability Density Analyzer was initiated there has been considerable discussion of probability density, as a mathematical concept and as a physical requirement. This discussion will therefore be brief. For a tutorial presentation we refer to Piersol[6] and Bendat et al,[7] and to Broch.[op cit] The following discussion introduces the notation which is used in this report.

Probability density and probability are not of the same dimension. Probability is dimensionless. It is usually referred to as "the ratio of the number of successes to the number of trials." It is not necessary to examine the traditional probability theory to establish the understanding necessary to discuss the probability density analyzer. It is more meaningful to define probability in terms of the actual quantities to be measured, that is, since in every case we are considering a random function of time we define probability as a set of measurements on such a random function of time.

Engineers refer to a random function of time as a random waveform, that is, for a given sample the amplitude can be plotted vs. the time. Examination of the graph of such a waveform reveals that on the average the signal appears to be in the vicinity of some amplitudes more often than others. Note that one must say vicinity of an amplitude for it is impossible to specify the amount of time spent at a given amplitude. When a signal takes on a specified value of the amplitude, the intercept of the waveform with a specified amplitude value defines only a point. The projection of this point on a time axis, defines therefore just a point in time and not a time interval. The sum of a finite number of points is of measure zero. It is this difficulty which requires the concept of probability

density. A large interval of time is chosen, T. An amplitude interval Δx is chosen. As one considers each amplitude x, the Δx defines the region about this amplitude. This then defines a segment of the waveform and in general the projection of this segment on the time axis defines a time interval.

For each x the time intervals are summed. The ratio of the sum of these time intervals to the total time is then defined as the probability that the signal is in the interval Δx at x. This is written as follows:

$$P\ (x \epsilon \Delta x)_T = \frac{\sum_i \tau_i(x, \Delta x)}{T} \qquad (1)$$

Note that this is a dimensionless number. It is the ratio of the sum of time intervals to another time interval. However, a difficulty is present in that there are two variables to the probability, the value of the amplitude x, and the value of the amplitude interval Δx.

The removal of the extraneous variable Δx leads to the concept of probability density. It is intuitive that the probability of something occurring in a small interval is in general proportional to that interval. For example, one would expect that if the average time that the signal was in a small interval Δx about the value x were p_1 then if we doubled the small interval to $2\Delta x$ that the probability would also be doubled. We would obtain a probability $P_2 = 2p_1$. It is therefore reasonable to assume that as long as the interval is very small, if we divide by the interval we obtain a quantity which is no longer dependent upon the interval, a probability function only of the amplitude. This

quantity will, of course, no longer be dimensionless. For example, if the amplitude x were the amplitude of a displacement then the result would be denoted as the probability per unit displacement. It is this quantity which we refer to as probability density.

Although we have pretended the probability will now only depend only on the amplitude our example indicated that a fundamental sample of duration T was chosen. The manner in which we eliminate this dependence will be discussed in the following section. It suffices here to say that as T becomes large, the ratio expressed in Eq. (1) ceases to become dependent on T.

Considering the range of values acceptable to the probability and to the probability density, we see from Eq. (1) that the probability must always be a number less than or equal to 1. This occurs because the sum of all possible sub-intervals in the total interval T can never be greater than T. This is not found to be true, however, when we examine the probability density which can now be given more precisely by:

$$w(x) = \lim_{\substack{\Delta x \to 0 \\ T \to \infty}} P(x \epsilon \Delta x) = \lim \frac{\sum_i \tau_i (x, \Delta x)}{\Delta x \, T} \qquad (2)$$

Here the probability density can take on any positive value or zero. However, we can restore the dimensionless character by multiplying the probability density by the infinitesimal dx. This quantity will always be less than or equal to unity. Of course, the sum over all the probabilities must now be replaced by an integral.

The rigorous probability density is clearly not accessible to direct measurement. This can be seen by the above equation.

1. It requires taking a limit as a quantity becomes infinitesimal.

2. It requires taking a measurement over a time interval which becomes infinite.

3. It requires a range of values which can extend to infinity.

For this reason the measurement of probability density must be an approximation, an estimate. The formula for the actual measurement defines a type of function which is usually called an "estimator." In the following section we will discuss the estimator used in the measurement of the probability density, and the consequences of using such an estimator. In general the estimator must give an answer which differs from the true analytic answer as described. The difference is an intrinsic error which must be evaluated and minimized in design.

For convenience in measurement evaluation the concept of normalization is introduced. The above indicates that one type of normalization requires that the probability density integrated over all values of the signal shall be unity.

To present data in a convenient dimensionless form another type of normalization is useful. This requires an understanding of the statistical quantity known as the standard deviation. Because the probability density of stationary random signals is being considered the standard deviation can be defined as follows:

(Standard Deviation) $\equiv$

$$\sigma = \lim \left[\frac{1}{2T} \int_{-T}^{+T} [x(t)]^2 \, dt - \left(\frac{1}{2T} \int_{-T}^{+T} x(t) dt \right)^2 \right]^{1/2} \qquad (3)$$

It is simply the square root of the difference between the average of the signal squared and the square of the average signal. A further normalization is now obtained by expressing the probability density in terms of a dimensionless quantity, $y = x/\sigma$.

III. THEORY OF MEASUREMENT OF PROBABILITY DENSITY

The discussion in Section I indicates that the probability analyzer actually evaluates an estimator for the probability density. This estimator has a meaning in terms of the waveform as indicated in Fig. 1. A feature of this estimator is that the limit as Δx approaches zero and as T approaches infinity is not taken. There are many consequences of this estimator not being precisely the equation for the probability density. Of prime interest is the fact that the estimator results in an uncertainty or a spread of error in the measurement. We characterize this spread of error by the normalized variance.[5]

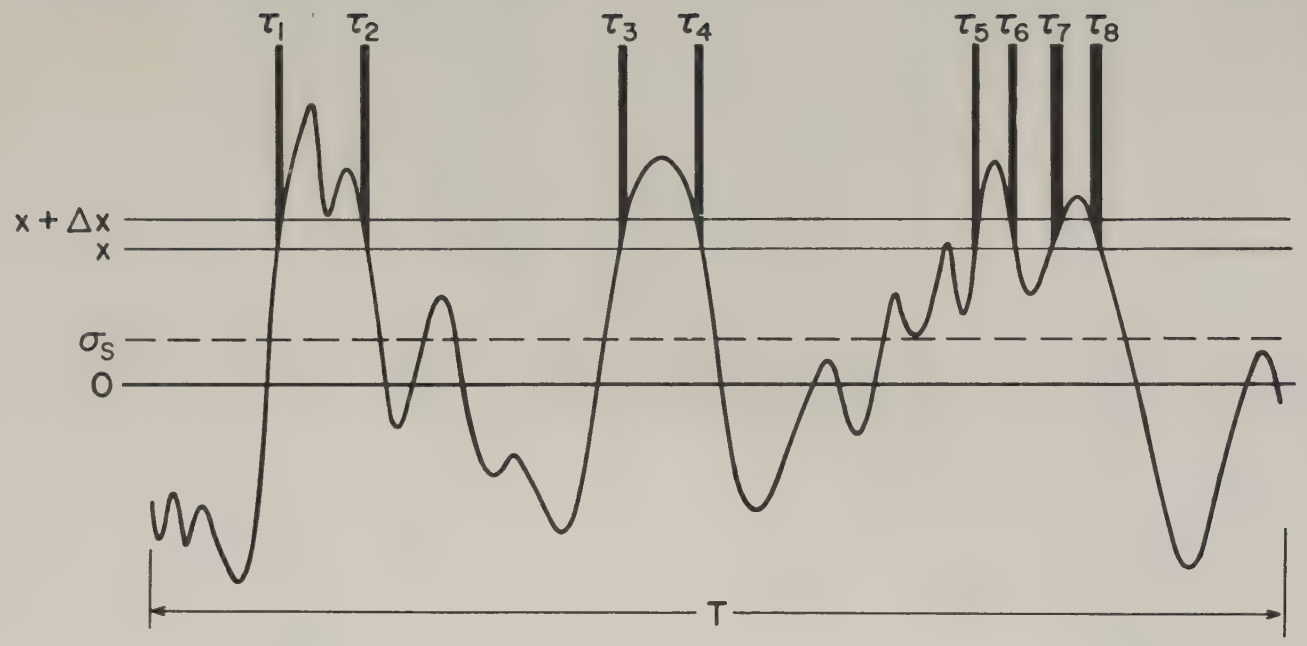
$$\sigma_n^2 = \frac{(3/8\pi)^{1/2}}{f_o T w(x) \Delta x} \qquad (4)$$

where f_o is the input "noise" bandwidth.

To use this normalized variance one must use the technique of confidence intervals of probability theory. Many tables are available for this purpose.[8] In Appendix A will be found examples of the use of this formula.*

It can be seen by examining the above equation that this does not completely specify the requirements on the probability density analyzer. We consider below two parameters which are very important in

* Alternate derivations yielding the same form but a different constant have been formulated by Bendat and Fox as reported in Bendat et al., op. cit.



ESTIMATOR

$$W_E(x) = \frac{P_E(x, x + \Delta x)}{\Delta x} = \frac{\sum_k \tau_k}{\Delta x T}$$

FIG. 1 GRAPHICAL ILLUSTRATION OF THE MEANING OF THE ESTIMATOR

specifying the performance of the probability density analyzer over and above the expression given for the normalized variance.

One can inquire as to how the error will vary for different values of x. It would be desirable that the error be constant for all values of the amplitude or be within a prescribed limit. To consider this question it becomes necessary to specify a form for the probability density. On the basis of physical arguments,* most probability densities describing vibration and sound data, are generally similar to the Gaussian distribution, $w(x) = (\exp -y^2/2)/2\pi$. Then, even as the time interval is taken to be infinitely long, the relationship between the estimator and the true probability density remains a function of the dimensionless variable y and the amplitude interval $\Delta y (= \Delta y/\sigma)$. This relationship can be expressed by a power series in Δy. The first terms are given below:

$$W(y)_{(\text{estimate})} = W(y)_{(\text{true})} \left[1 + \frac{y^2-1}{24} (\Delta y)^2 + \ldots\right] \qquad (5)$$

The required window width can be determined by noting the rapid convergence since Δy is considered small. One chooses a maximum value of y which is of interest and a maximum acceptable error in the estimator for the probability density and then solves for Δy.

There is another consideration which is more subtle. Two different random signals may show the same probability density, that is to say their estimator may have the same value. Yet the duration of a typical contribution to the average time spent at a given interval

* The applicability of the "central limit theorem".

may be very different for the two signals. If the signal varies very rapidly it may pass through the interval Δx in a very short time, whereas if it varies slowly the time can be several orders of magnitude longer. In a practical instrument there will be some limit which describes how brief a transversal of the amplitude window will be registered. We denote this as the "time resolution". This time resolution is dependent on the bandwidth, f_o and the maximum dimensionless amplitude that is to be measured, y_o. The expression is

$$\text{(time resolution) } \tau_o = \frac{\Delta y}{2\pi y_o f_o} \qquad (6)$$

IV. PROBABILITY DENSITY ANALYZER - GENERAL THEORY

In this Section we present how instrumentation concepts can be formulated to realize the logical operations implicit in the formula for the estimator of the probability density [Eq. (2)].

A. Logical Analysis - Operations

The implicit logical operations are:

1. Normalization:
   To express all results in terms of a dimensionless amplitude is equivalent to scaling the amplitude of the random function of time to a standardized level.

2. The Independent Variable:
   Since the probability density is to be a function of the dimensionless amplitude y it is necessary to establish a method whereby the circuit creates a unique condition corresponding to the signal equaling the "set-in" amplitude. A convenient way to accomplish this is to combine the "set-in" amplitude value with the signal in such a way that the combined signal and amplitude value will yield a unique value (say 0) whenever the signal is equal to the "set-in" amplitude.

3. The Amplitude Interval:
   The equation for the estimator has as a parameter Δx a specific amplitude interval. It is necessary therefore to establish circuitry such that (2) above is amended to indicate a unique value not merely for the signal equal to the "set-in" amplitude, but for the signal in a region about the "set-in" amplitude defined by the amplitude interval Δx.

4. Measurement of the Time Intervals Corresponding to the Amplitude Intervals:
   The τ_1 in the equation for the estimator are the individual time intervals which are the projection of the amplitude intervals on the time axis. These must be summed. It is therefore necessary to generate signals which correspond to the time durations so that they may be suitably summed.

5. Averaging - Summation and Division by T:
   The signal developed in step (4) must be averaged. Furthermore, some method must be included for performing the equivalent of dividing by the total time interval of the measurement.

6. Division by the Amplitude Interval Δx:
   The one remaining operation is to divide by the amplitude interval to obtain the proper dimensions for the probability density.

The above six steps are indicative of what any probability density analyzer must do in some way or other.

B. Functional Analysis - Waveforms

In Fig. 2 a block diagram is illustrated which indicates the signal processing which is required to measure probability density. Each box represents one of the functions indicated in the above six logical steps which describe the formula for the estimator. In addition to the random signal input and the probability density output another output is indicated as the "amplitude identification output". This is required in those cases in which the selection of

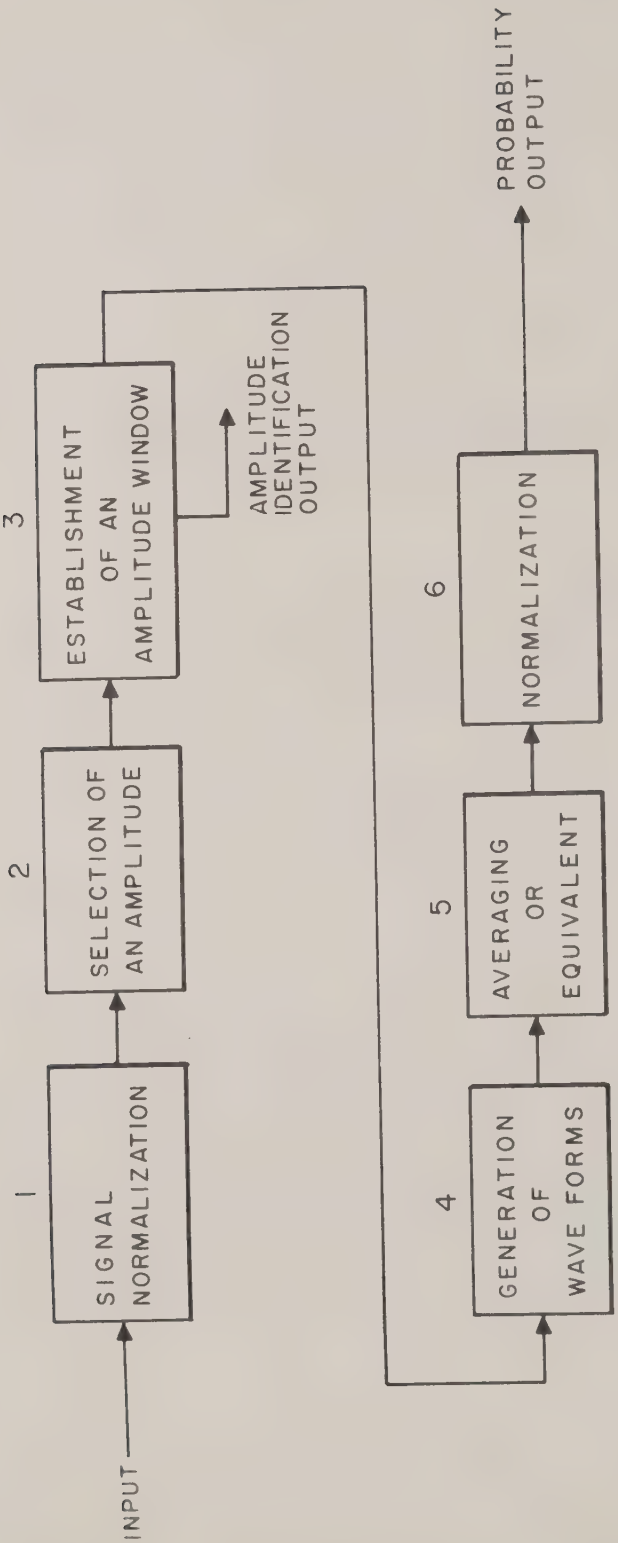


FIG. 2 SIGNAL PROCESSING REQUIREMENTS TO MEASURE PROBABILITY DENSITY

the amplitude is an internal function of the instrument. It is then necessary to provide information to the user as to which amplitude has been selected.

This signal processing can be represented also by a sequence of waveforms. Such a sequence is presented in Fig. 3. Figure 3a and 3b indicate Step 1 - the signal normalization or scaling. In a and b are indicated a standardized reference RMS value. a being the input waveform, can have an actual RMS which differs from this. The simple function of the first block in the signal processing Fig. 2 is to scale the signal illustrated in a until its actual RMS corresponds to the reference RMS as indicated in b of Fig. 3.

Sections c, d and e of Fig. 3 indicate the waveforms corresponding to the selection of a specific amplitude. In c the waveform is again illustrated and a level is indicated as an example of a "set-in" amplitude value. As was indicated previously a convenient way to identify this amplitude would be to cause the signal to have the unique value of zero when the signal is equal to the "set-in" amplitude. This is illustrated in d of Fig. 3. The operation amounts to an axis shift such that the signal is shifted until the "set-in" amplitude values coincide with the zero axis. In e is an enlarged drawing of a segment of the waveform d as indicated.

The processing required by block no. 3 of Fig. 2 is illustrated in e, f and g of Fig. 3. In e of Fig. 3 the "set-in" amplitude value has been identified as the new zero crossing of the waveform but an amplitude interval, Δy has not been established about this level. A manner in which this can be accomplished is indicated by the following two waveforms. In f of Fig. 3 the negative of the absolute

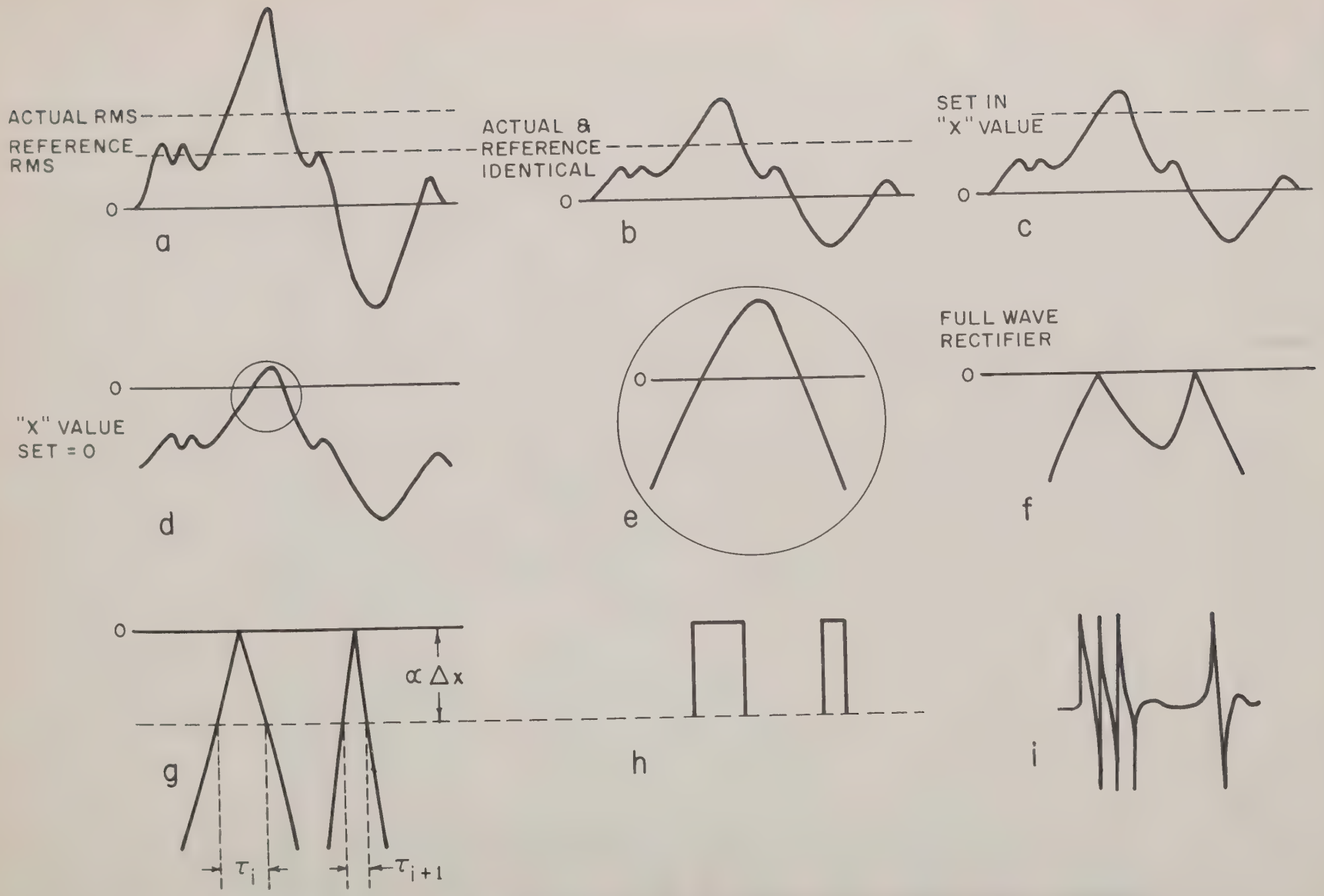


FIG. 3 SEQUENCE OF WAVEFORMS REPRESENTATIVE OF THE SIGNAL PROCESSING AS INDICATED IN FIG. 2

value of e is taken. The specified amplitude is now unique in two ways. It is uniquely zero and it is also the maximum. It has been indicated that the amplitude interval Δx is very small. More specifically it should be very small with respect to the "set-in" amplitude itself. This is illustrated in g of Fig. 3. The waveform of f is greatly magnified. Because the absolute value of the deviation of the signal from the "set-in" value has been taken, both positive and negative deviations from the "set-in" value are indicated as excursions below the zero axis as given in g of Fig. 3. It can be seen that the interval between the zero axis and the amplitude level below it is proportional to an amplitude symmetric about the "set-in" x level. The proportionality will simply be related to the amplification that was used to obtain the signal g.

Block no. 4 of Fig. 2 now requires the generation of waveforms proportional to the time interval corresponding to the amplitude interval. Illustration g is an indication of the relationship of the amplitude to the time intervals. The projections of the intercepts of the waveform with the amplitude intervals are the time intervals τ_i. h of Fig. 3 illustrates suitable waveforms which satisfy the criterion of being proportional to the time interval. They are proportional in width and because they have constant amplitude they are proportional in energy also.

It remains now to average the time intervals, to take the sum of the time intervals and divide by the total time. One of the many ways in which this can be accomplished is indicated in illustration i of Fig. 3. The waveforms of h are used to gate a clock. The clock then operates a totalizer which accomplishes the summing.

The division by the total time is a scalar operation which is not easily illustrated. Similarly the operation of block 6 of Fig. 2 of normalization by the division of the amplitude intervals is a simple scalar operation when the amplitude interval is fixed.

C. Functional Analysis - Block Diagram

In Figure 4 is presented a block diagram of the B + K model 160 Probability Density Analyzer. By cross reference to Fig. 2 and the waveforms of Fig. 3 the function of the various blocks can be understood. The process of the signal normalization is accomplished in the input circuits by means of a meter with a fiducial calibration point. The input circuits also accomplish function 2 the selection of an amplitude. As indicated in Fig. 4 the potentiometer selects the value of the amplitude and applies it to the input circuits at the same time making the amplitude signal available externally for plotting purposes. In the input circuits this is added to the signal in opposite polarity so that when the magnitude of the signal and the reference amplitude are equal a zero is produced at the output.

The waveforms indicated in Fig. 3 illustrations e through g are achieved through a differential heterodyne absolute value circuit. This circuit, the Arguimbau circuit will be discussed separately.

The block labeled "Gate Generator" produces an output waveform corresponding to illustration h of Fig. 3. The remaining blocks in Fig. 4 are various methods of producing the output. The 10

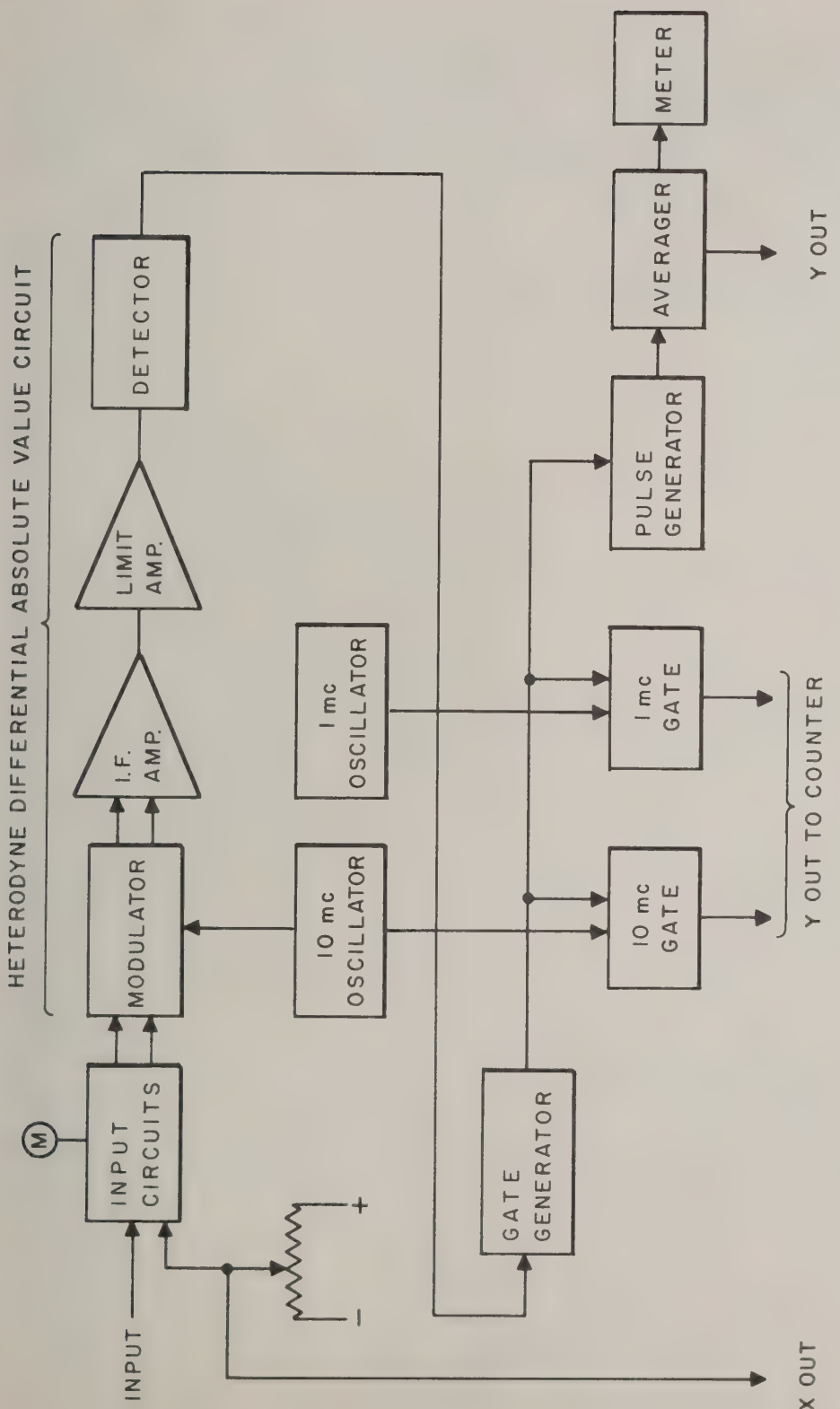


FIG. 4 FUNCTIONAL BLOCK DIAGRAM OF B&K PDA MODEL 160

megacycle gate and 1 megacycle gate produce waveforms similar to the illustration i in Fig. 3. The remaining three blocks produce a direct voltage average which is obtained by averaging the pulses as illustrated in h of Fig. 3 with an RC averaging circuit. This is available either as an output voltage or as a meter reading.

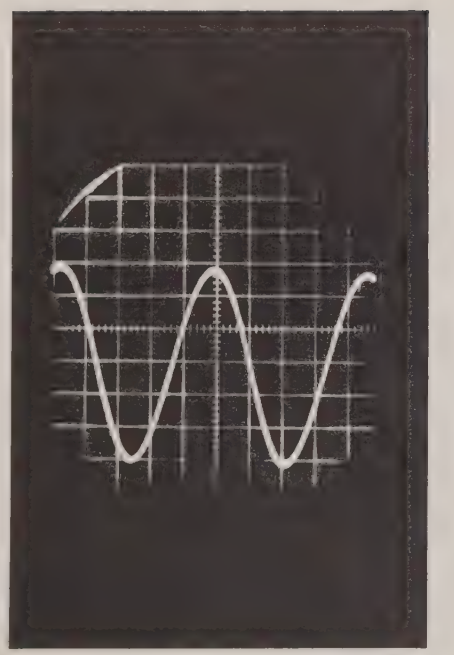
A detailed discussion of the workings of these circuits is provided in Part B of section VI.

V. USE OF THE PROBABILITY DENSITY ANALYZER

A recent report [Bendat et al, op. cit] discusses the manner in which a probability density analyzer can be used in a general data reduction system. In this section some examples of the use of the PDA in physical studies and control measurements are presented.

It is the nature of a statistical measurement that it is comparatively insensitive to the details of the signal. In spite of this characteristic it is not always clear what constitutes a detail and what constitutes an essential facet of the signal. For this reason there are many investigations of information containing signals which can be obtained and measured by statistical analysis. This is of particular interest in sonar and radar. An example of how the probability density analysis of a waveform brings out a sublety is illustrated in Fig. 5.

The first illustration is of the waveform obtained from a Hewlett Packard 202 low frequency function generator. The waveform appears entirely sinusoidal. Visual observation indicates no essential departure from a sinusoidal waveform. A probability density analysis is illustrated also. For this particular application an oscilloscope was used for the readout device. This probability analysis departs from what one would expect as the probability density of a sine wave (see Appendix B) in the rectangular like structure of the probability density curve. The third oscillograph presents the same signal differentiated. The sinusoidal output of the Hewlett Packard 202 function generator is obtained from a triangular wave which is passed through a segmented function circuit. This illustration indicates how a comparatively subtle feature of the waveform may turn up in probability density analysis.



HEWLETT PACKARD 202 OUTPUT

PROBABILITY DENSITY

HEWLETT PACKARD 202 OUTPUT
DIFFERENTIATED

FIG. 5 THE SENSITIVITY OF PROBABILITY DENSITY ANALYZER TO CERTAIN TYPES OF WAVEFORM DISTORTIONS - EXAMPLE

Another use of probability density analysis concerns the problem of fatigue and the non-linear response of mechanical structures. If for one excitation a structure of a certain material receives a high stress level less frequently than for another excitation the fatigue lives will differ in general. When the source of excitation is random the distribution of stress levels may depend not only on the source but on non-linearities. An illustration of the behavior of extreme statistics with a non-linear parameter in a single-degree-of-freedom system is given by Fig. 6 taken from a paper by Smith and Lambert[9]. These curves emphasize the importance of precise measurements. The deviations of the probability density curves to linear and non-linear response are quite small but yet these deviations can be critical as to whether a structure will fatigue.

Another example of the use of probability density analysis is demonstrated in the illustrations of Fig. 7[10]. These curves exhibit the results of a study concerning the statistics of an excited single-degree-of-freedom system when the excitation is by a sweep frequency. This kind of information can have a variety of applications. In this particular study the interest was in methods of simulating random Gaussian noise for structural fatigue studies. However, the same kind of analysis would be useful in sonar and radar work to determine how the statistics of multiple doppler shifts from many moving single frequency sources would differ from other types of noise.

Another interesting use of probability density analysis is also concerned with signal information, the handling of telephone signals on transmission lines. An example is shown in Fig. 8. This data was obtained by Slack[11] in the study of the overloading

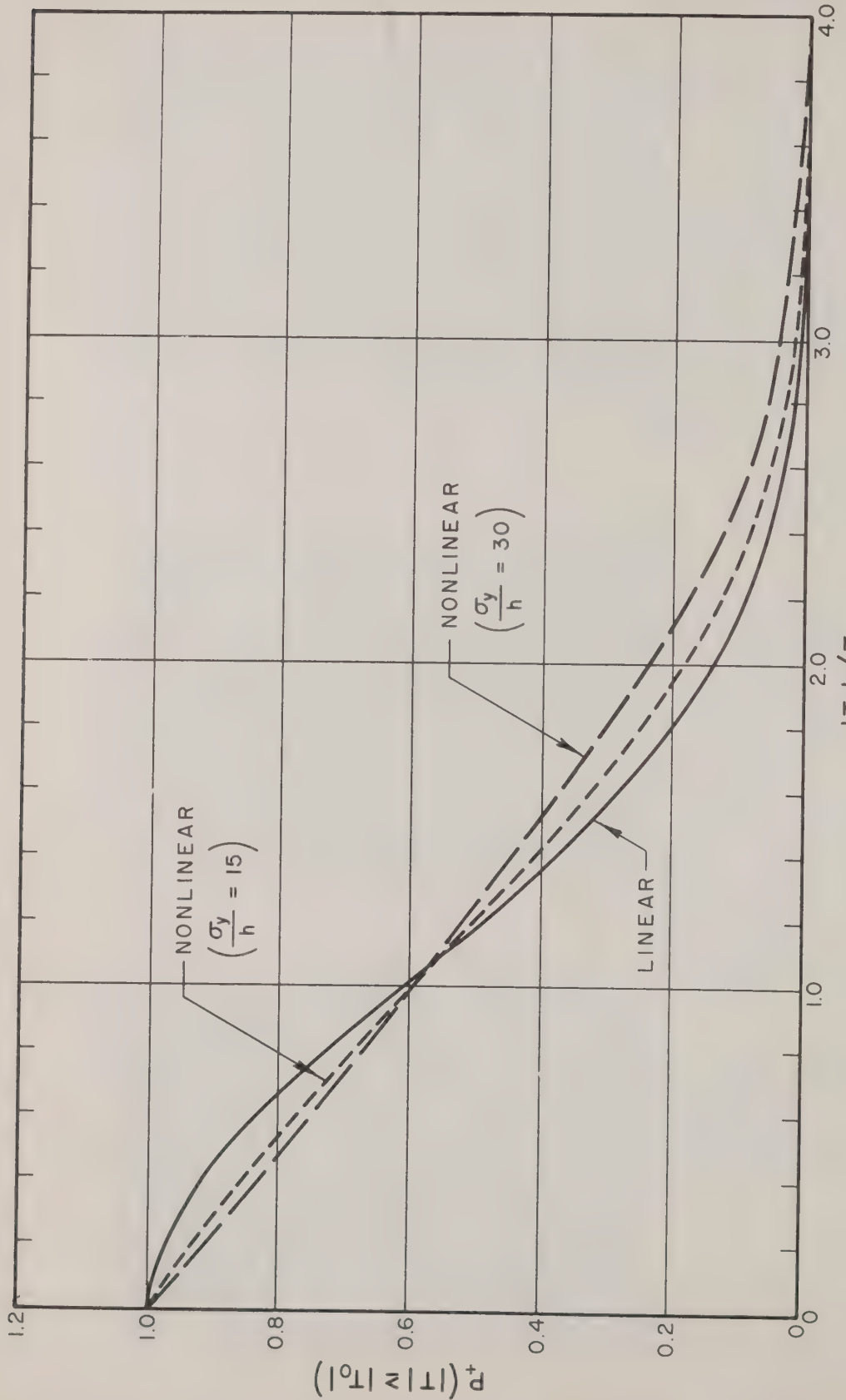


FIG. 6 PROBABILITY DISTRIBUTION OF STRAIN MAXIMA FOR SEVERAL EXCITATION LEVELS OF SINGLE DEGREE OF FREEDOM SYSTEM (AFTER SMITH AND LAMBERT)

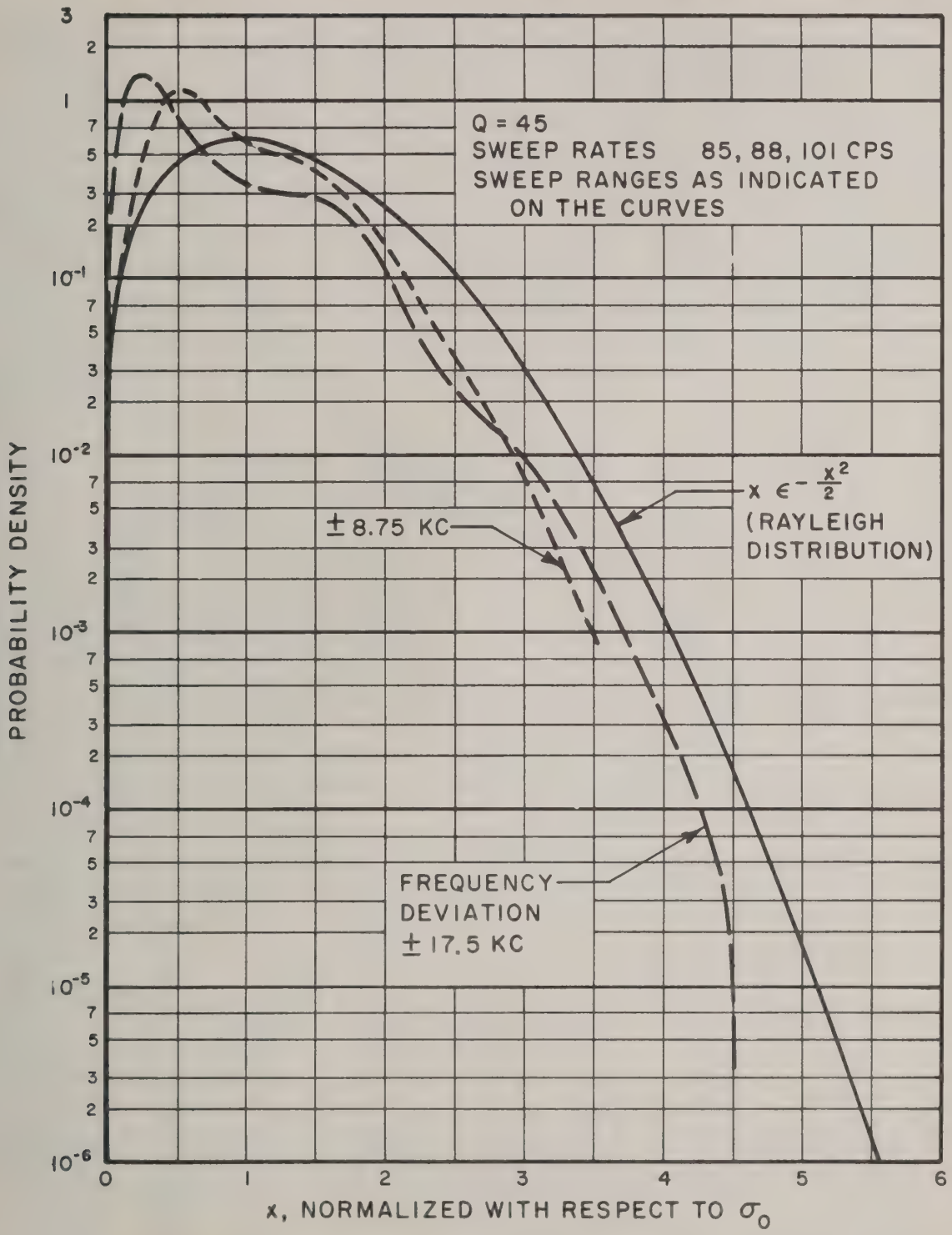


FIG. 7 PROBABILITY DENSITY CURVES FOR THE ENVELOPE OF THE RESPONSE OF A TUNED CIRCUIT TO THREE SUPERPOSED SWEPT FREQUENCIES (AFTER ARGUIMBAU, BBN REPORT NO. 685)

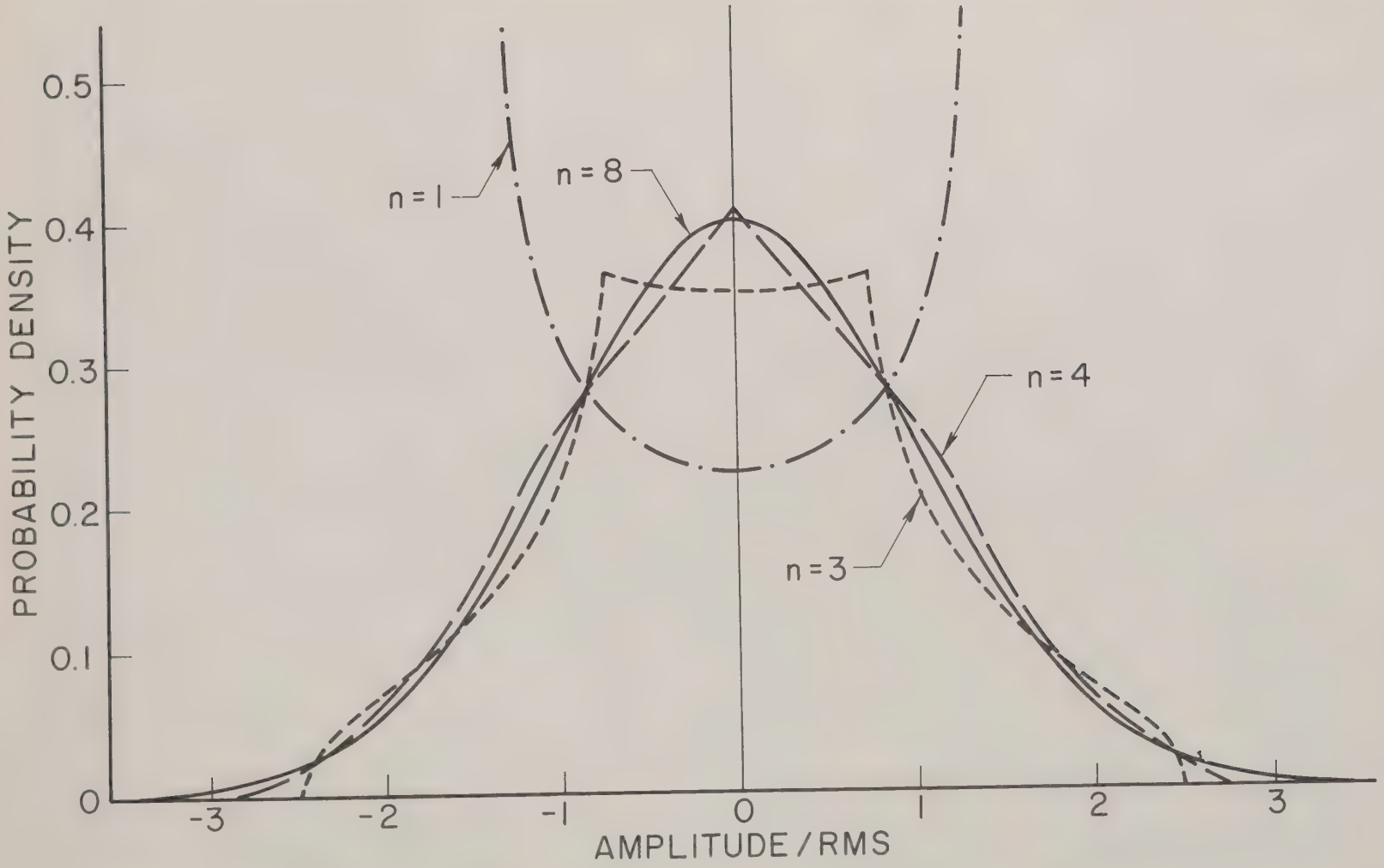


FIG. 8 THEORETICAL DISTRIBUTION OF INSTANTANEOUS VALUES OBTAINED FROM THE COMBINATION OF n COSINE WAVES. (AFTER SLACK)

of telephone transmission lines. It illustrates the distribution of instantaneous values obtained with the combination of many cosine waves of the same amplitude. Calculations for more complicated cases are extremely difficult and in such cases a probability density analyzer is indispensable. For example, in Fig. 9 are comparisons of the probability densities of the sum of two cosine waves which are not of the same amplitude[12]. These were obtained by measurements with the PDA. Further illustrations of the wide diversity of probability density curves that can be obtained experimentally are given in Figs. 10 and 11.

Some other uses of probability density analysis are listed below:
(1) In the study of electroencephelographs ... Work of this nature is being done at present at BBN by Dewey Neff and co-workers.

(2) Studies of temperature fluctuations to extablish the stability and instability of certain systems ... An example is in a reactor pile. Such studies have been made at Oak Ridge National Laboratories.

(3) The study of hydrodynamic and micrometerological fluctuations ... Work of this type has been done under NSF Grant G-4542.

(4) Studies for determining the essential information carrying properties of speech waveforms ... Work of this type is being carried on at Cambridge AFCRC and elsewhere.

The statistical nature of certain signals is also of interest in studies of human operator characteristics, for example work done at BBN by J. I. Elkind.

The nature of the propagation in a reverberant field can very well be examined statistically as illustrated in Fig. 12[13].

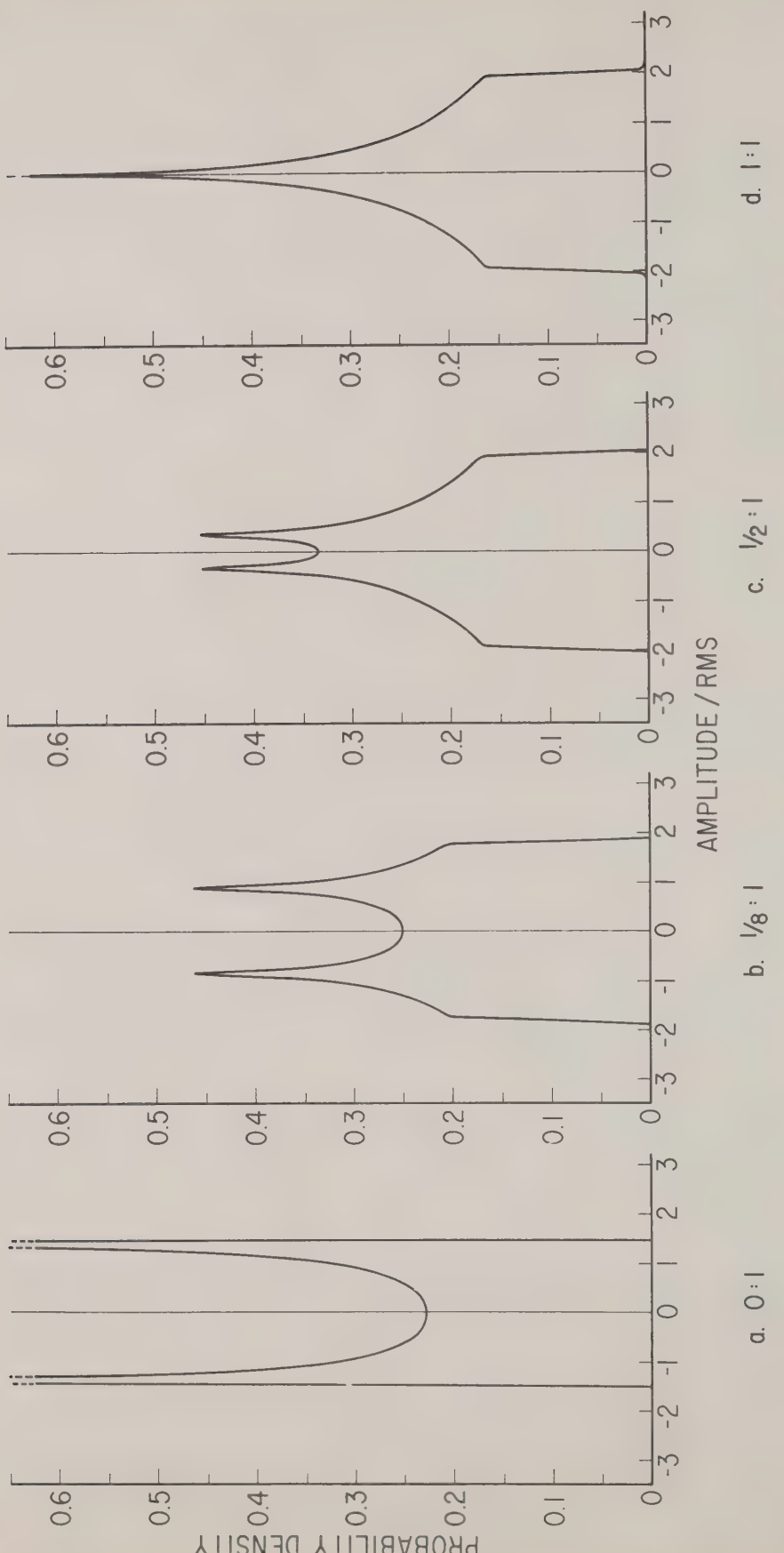


FIG. 9 PROBABILITY DENSITIES REPRESENTING THE SUM OF TWO INDEPENDENT COSINE WAVES FOR FOR VARIOUS ENERGY RATIOS. (AFTER FOX AND STARR)

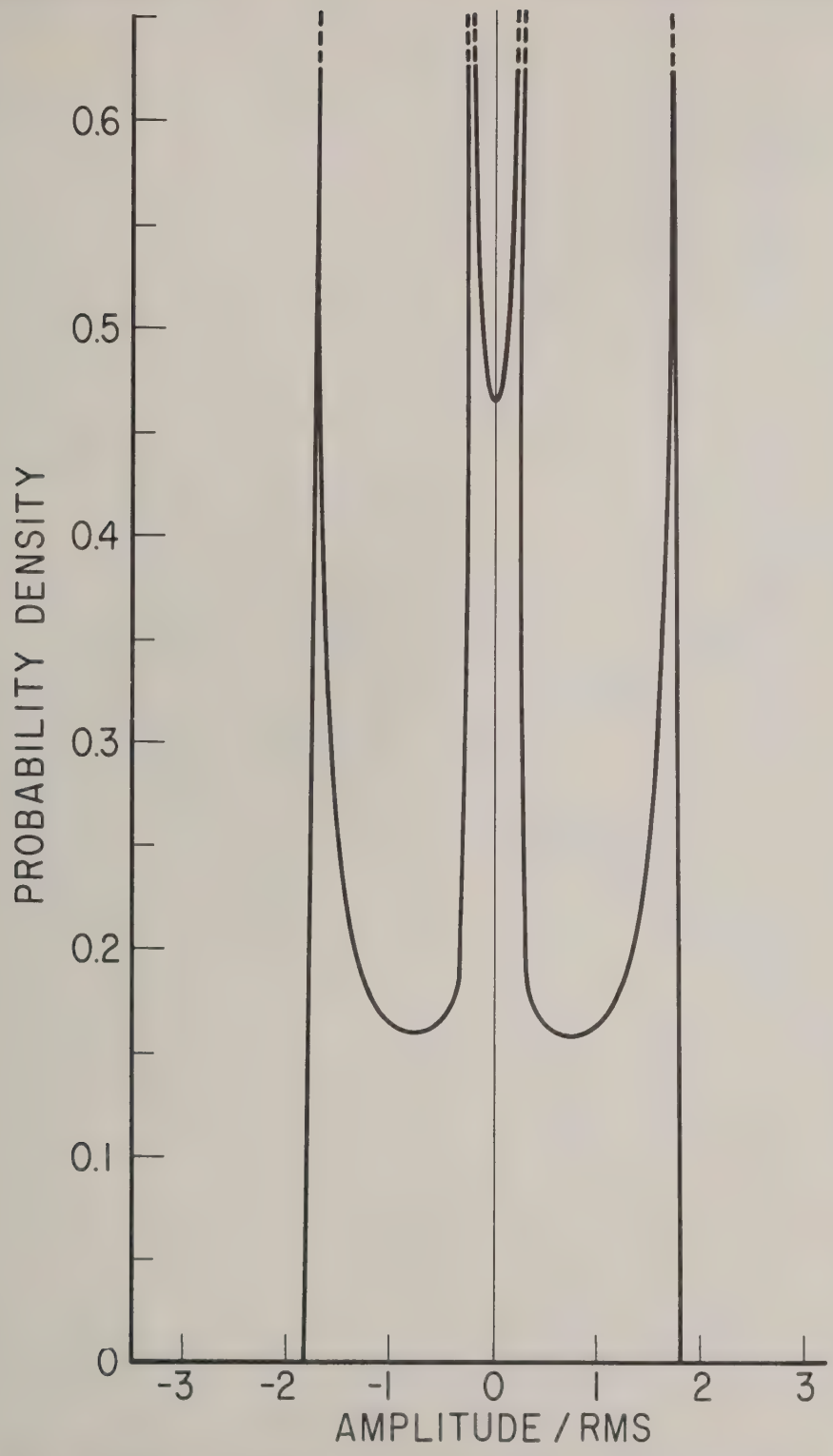


FIG. 10 PROBABILITY DENSITY OF THE SUM OF INDEPENDENT SQUARE AND SINE WAVES FOR EQUAL ENERGY RATIOS. (AFTER FOX AND STARR)

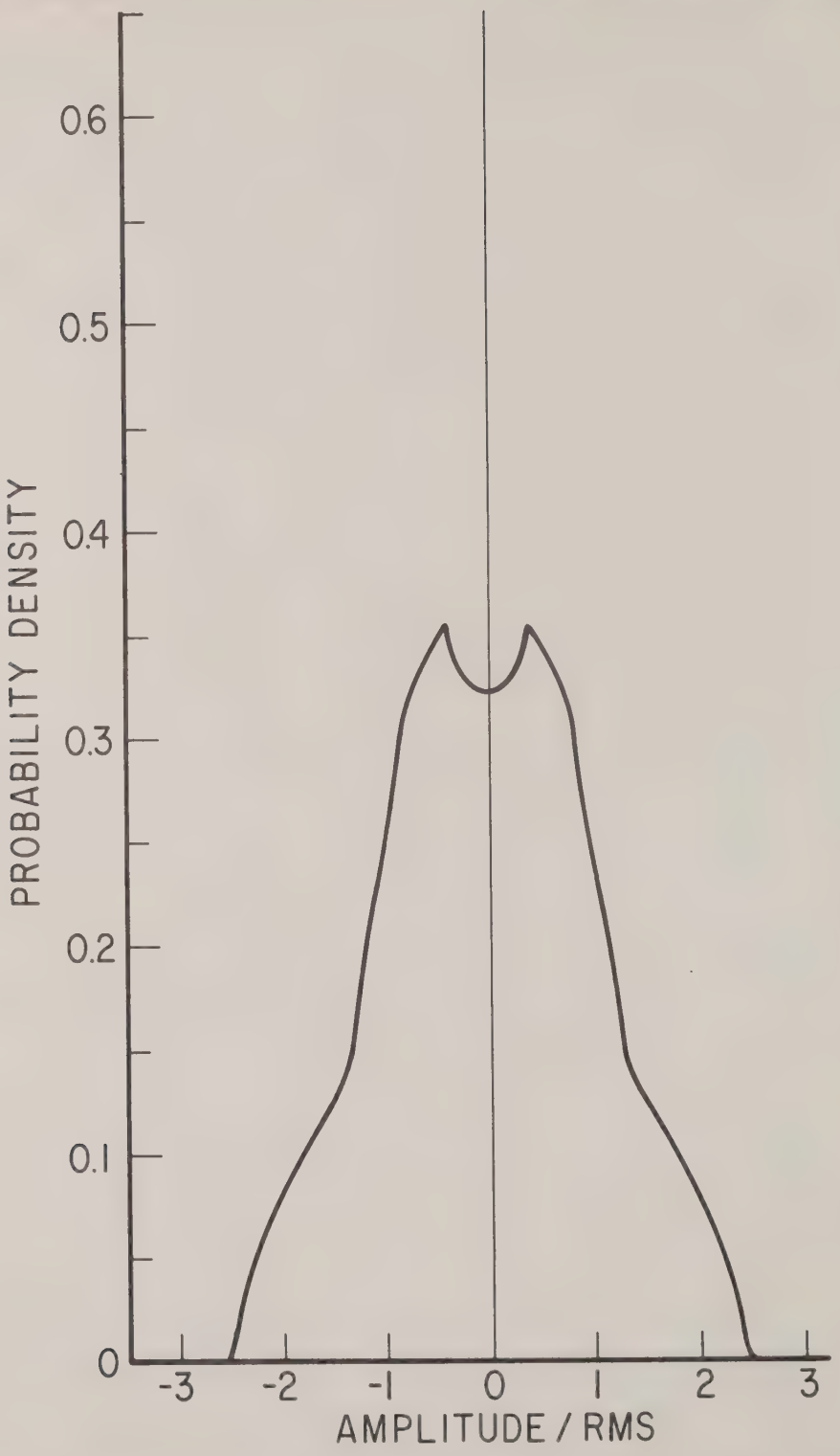


FIG. 11 PROBABILITY DENSITY OF THE SUM OF INDEPENDENT SQUARE, COSINE AND TRIANGULAR WAVES FOR EQUAL ENERGY. (AFTER FOX AND STARR)

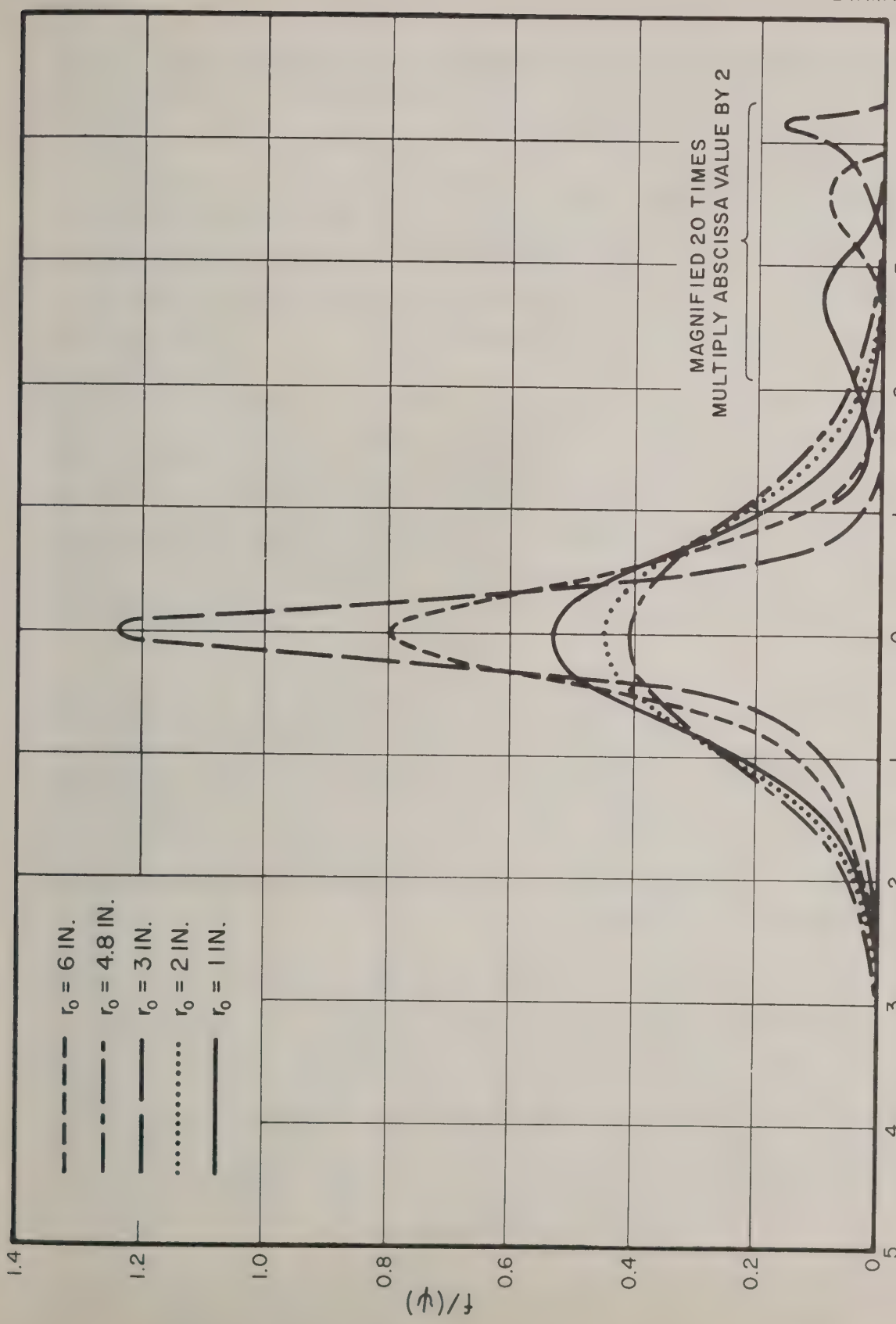


FIG. 12 PROBABILITY DENSITY OF PRESSURES AT VARIOUS DISTANCES FROM A PULSED SOURCE IN A "HARD ROOM" (AFTER LYON)

The use of a probability density analyzer in a complex data processing system is also of interest. Some applications have been discussed in the work of Bendat et al (op. cit).

Further refinements using more than one statistical analyzer are also of interest. Such a system can be used for obtaining joint probability and conditional probabilities, for both ensemble and time statistics. These can be very useful in propagation studies and studies on the nature of interacting modes in vibrating systems.

As a final example of the use of the probability density analyzer we consider the assessment of the characteristics of a noise generator. For many purposes a random noise generator is required in electrical, acoustical, and mechanical measurements. Often, it is desirable that the output of the noise generator be a waveform whose amplitudes are distributed according to the Gaussian or normal distribution. (Other distributions may be desirable such as that of the random telegraph signal[14].)

A probability density analyzer can be used in such cases to measure the output waveform. Typically the output of a commercial noise generator which obtains its noise from a gas-tube is not distributed symmetrically. As an example the output of a widely used commercial noise generator has been analyzed. The probability density curve is shown in Fig. 13. Comparison of this curve with the Gaussian curve clearly displays an asymmetry.

Another noise source which has come into use recently is that of the noise generated in certain semiconductor diodes. In Fig. 14 an analysis of such noise is presented. A method for considering cases such as these where the probability density is asymmetric but otherwise Gaussian or deviates in some other from a Gaussian distribution is considered in Appendix C.

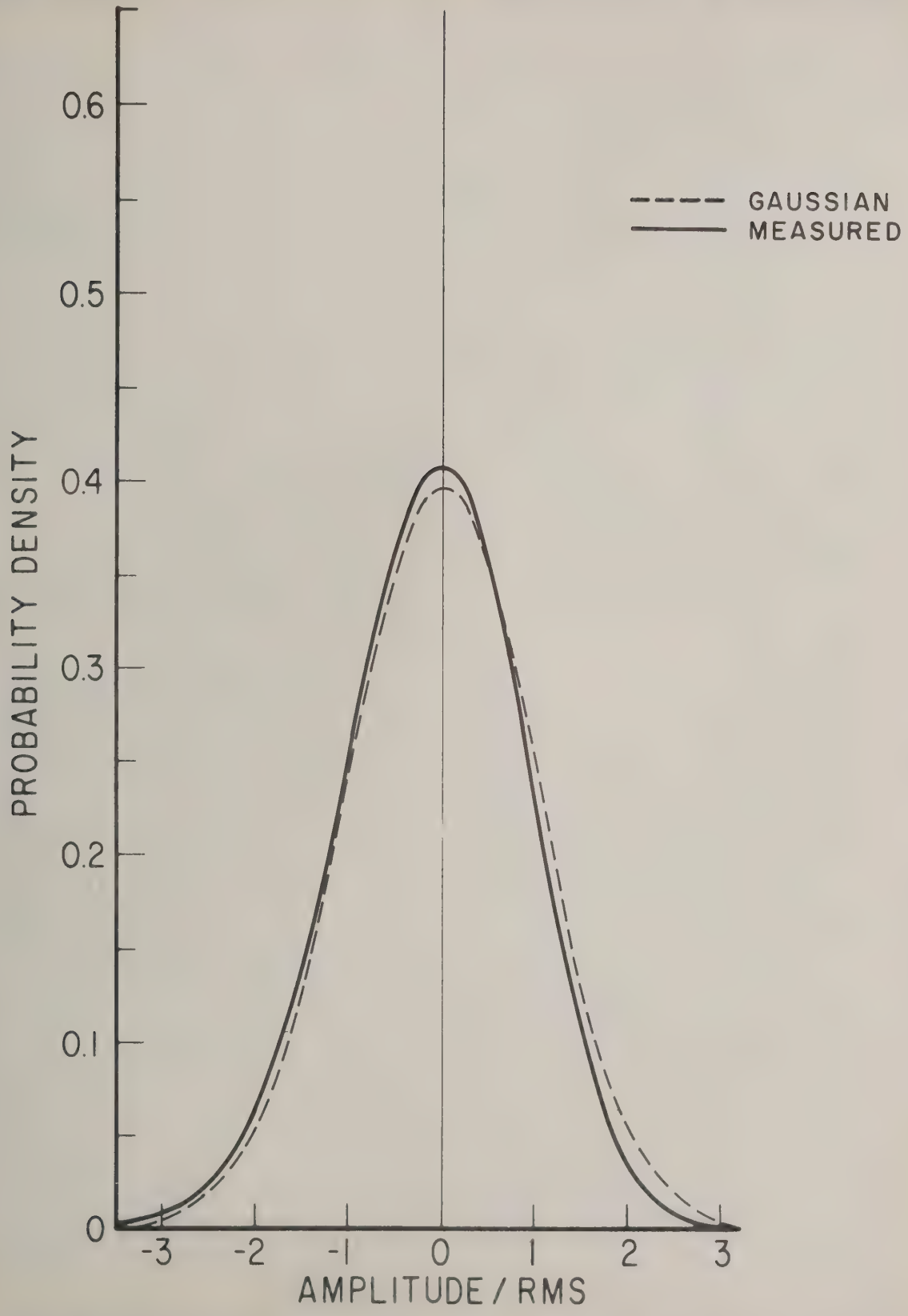


FIG. 13 GAS TUBE NOISE SHOWING NON-GAUSSIAN CHARACTERISTIC

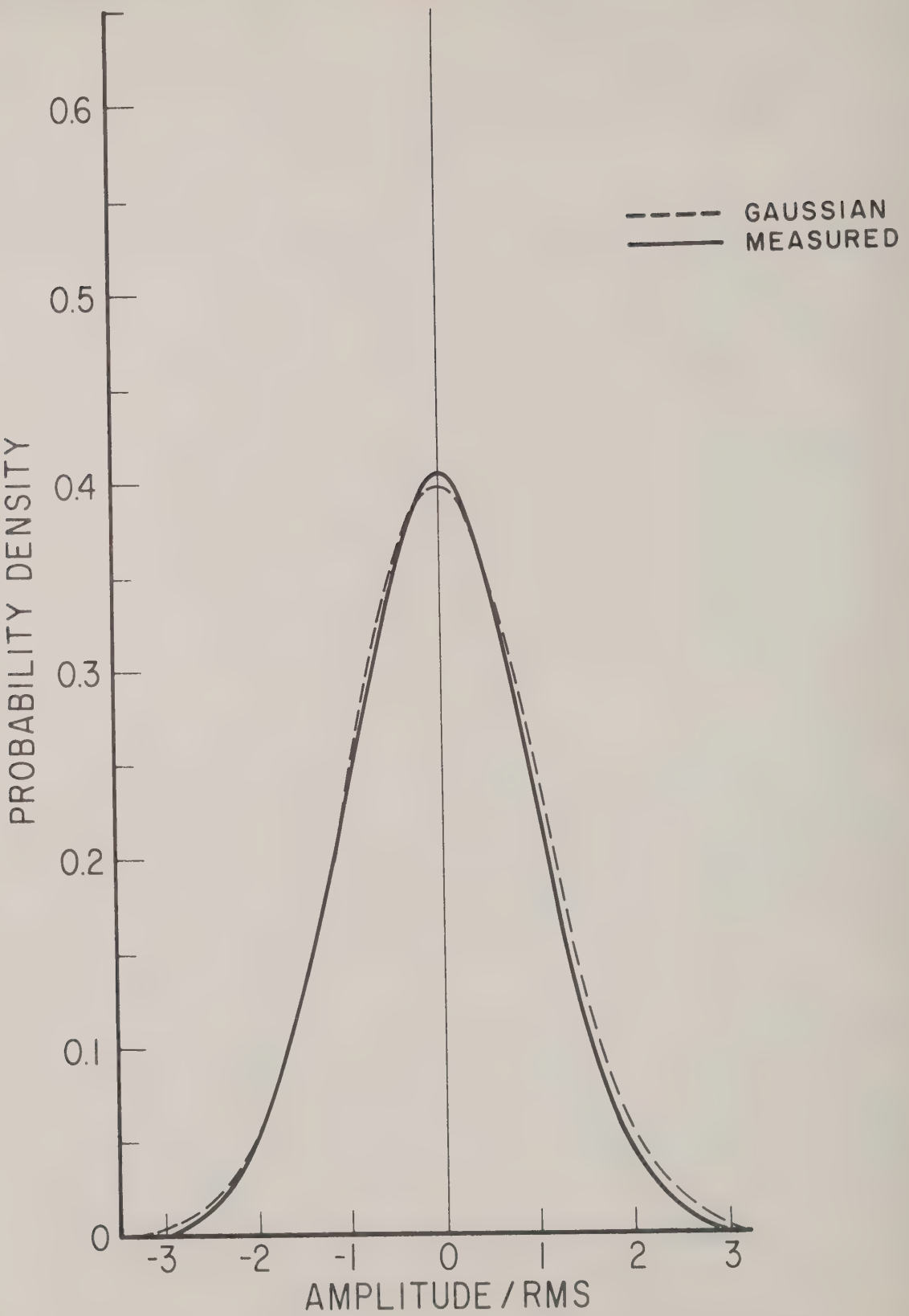


FIG. 14 SILICON DIODE NOISE SHOWING NON-GAUSSIAN CHARACTERISTICS

VI. THE B + K MODEL 160 PROBABILITY DENSITY ANALYZER -- DETAILED DISCUSSION

The material in this section supplies detailed information about the B + K Model 160 Probability Density Analyzer. This material refers to the Model B which is the improved version now being manufactured by CIC for B + K Instruments.

The material is broken up into five parts. In Part A, Operational Procedures are presented for calibrating and using the PDA in a variety of measuring situations. In Part B a Circuit Analysis is presented. In Part C an Alignment Procedure is outlined. In Part D Performance Data is presented. This performance data was obtained on the first two instruments of the Model B improved version and are the basis for the present production run. On the basis of this performance data, conservative specifications have been established and are presented in Part E.

A. Operational Procedures

A variety of operational procedures can be devised depending on the measurements which have to be made, the type of accuracy required, and the type of read-out equipment to be used. The calibration technique and the method of operation for each of these situations varies. There are however many adjustments and procedures common to these techniques. The first procedure presented below is given in considerable detail. It is the calibration and use procedure required when the PDA is to be used for simple measurements with no additional read-out equipment. Reference to this general procedure is made throughout the remaining presentation of procedures.

The first five operational procedures presented are distinguished one from another by the type of readout equipment they use. The first procedure refers to the use of the PDA by itself. The remaining procedures consider the use with an X-Y pen recorder, with an external sweep and display device such as an oscilloscope, with a digital counter, and with a B + K 2305 graphic level recorder.

The operating procedures 6 through 10 are techniques of using the instrument to obtain measurements other than the simple probability density of the amplitude of the input function. Procedures 6 and 7 which present methods of obtaining the exceedances and methods of obtaining one form of the distribution of the peaks are operational procedures for using additional outputs available on the probability density analyzer. Procedures 8, 9, and 10 however concern the use of the PDA not only with additional readout equipment but with additional input equipment.

The operational procedures 11 and 12 describe the use of the PDA in situations where the normalizing feature is not desired or not required.

1. General Procedure

The PDA is capable of a high degree of precision when used properly. The following instructions, when carefully followed should help to insure proper use. However, there is no substitute for a good understanding of the principles of operation of the instrument. It is suggested therefore that the general theory as outlined in section IV and the detailed theory, Part D of this section be read and understood before one undertakes use of the instrument. If the function of each instruction for calibrating and using the

instrument is understood, the operator will be better able to make the necessary adjustments in the operating procedure to satisfy the unique requirements of his measurement program. It is especially to be emphasized that interpretation of the specification in terms of an understanding of the theory of operation will permit the instrument to be used beyond its specifically stated capabilities.

a. Calibration

BEFORE TURNING THE INSTRUMENT ON ...

Place a shorting cap over the INPUT jack.
Place the input ac/dc switch in the DC position.
Rotate the input potentiometer clockwise to full scale.
Set the METER SCALE switch in the center SHORT position.
Rotate the OUTPUT-DAMPING-SWEEP TIME switch full clockwise to position G.
Set the SWEEP switch in the center position marked OFF.
Set the "X" potentiometer to zero.

NOTE

There are two zeros. The black "0" is the true zero.

Rotate the FUNCTION switch full counter clockwise to the position C-R BAL.

NOW TURN THE POWER ON ...
It is wise to allow the instrument to warm-up for at least ten minutes. When possible a warm-up time of twenty to thirty minutes is preferable.

ADJUST THE BALANCE CONTROLS ...
Pull the front drop-out panel open; it is hinged on the bottom.
Observe the position of the pointer on the PROBABILITY DENSITY meter.
Adjust the controls marked R and C. Obtain a null reading on the PROBABILITY DENSITY meter. A careful adjustment can bring the meter almost to zero.
Rotate the FUNCTION switch one detent to the position labeled DC BAL.
Adjust the center calibration control behind the drop-out panel labelled DC for a null on the PROBABILITY DENSITY meter. This can also be adjusted to a reading of almost zero.

If a very low reading is not immediately obtained on the PROBABILITY DENSITY meter, then repeat the last two adjustments. First adjust the R and C controls while the FUNCTION switch is in the C-R dial POSITIONS, then adjust the DC BAL. positions. When a suitable null has been achieved, proceed to the next step.

ZERO THE OUTPUT ...
Rotate the FUNCTION switch to the position marked METER-ZERO.
Push the METER SCALE switch to the left to the position labeled 0.4 (CAL).
Adjust the METER-ZERO control until an exact reading of zero is obtained on the meter.

CALIBRATE THE AMPLITUDE WINDOW ...
Return the METER SCALE switch to the SHORT position.
Rotate the FUNCTION switch to the CAL position.

Rotate CAL-SET fully counter clockwise.
Depress the push-button RMS METER located below the INPUT METER. The reading should be essentially zero.
While maintaining the button depressed, rotate the CAL control clockwise until the meter indicates precisely on the red line labeled CAL.
Release the RMS METER push-button.
Push the METER SCALE switch to the right momentarily.
If the PROBABILITY DENSITY meter reads on scale the METER SCALE switch may be left in the right position. Otherwise the GAIN should be rotated counter clockwise until this condition is achieved. Note the meter reading.
Push the METER SCALE switch to the left to the position labeled (CAL).
Adjust the GAIN to obtain a meter reading of 0.4 (full scale).
Close the drop-out panel.

If the meter reading was not within .35 and .45 then after adjusting the GAIN as indicated, the entire calibration procedure should be repeated before continuing. The reason for this procedure is to minimize the effects of interaction of the gain with the balance controls when the gain is far from the correct value. When the gain is finally adjusted, proceed.

CALIBRATE THE INPUT LEVEL...
Place the METER SCALE switch in the SHORT position.
Remove the shorting cap from the INPUT jack.
Connect with a BNC Amphenol 31-350 connector to the INPUT jack from a signal source suitable to the 10,000 ohms input impedance. Switch the AC-DC to the position desired. In the AC position an effective low-frequency cut-off of 50 cycles is present.

Place the FUNCTION switch in the USE position.
Rotate the input potentiometer at the upper left-hand corner of the instrument fully counter clockwise.
Depress the RMS METER button momentarily. If the INPUT METER reads up scale then the input signal is too large for the instrument. The indication should be essentially zero.

NOTE

The INPUT METER uses a thermocouple. DO NOT maintain the RMS METER push button depressed if the meter reads off scale.

Maintaining the RMS METER button depressed, rotate the input potentiometer clockwise until the INPUT METER indicates at the fiducial mark on the upper part of the scale. Set as accurately as possible.

The instrument is now readv for use. The above procedure has made use of the instruments internal calibration. For very precise work, other methods of calibration may be desirable. These are discussed in Appendix D.

b. Use

MEASURE THE INPUT RMS ...
The measurements obtained with the PDA refer to a dimensionless amplitude scale. If it is desired to determine the RMS voltage at the input for labeling the data, take the reciprocal of the reading of the input potentiometer. Full scale (clockwise,

ten-turns) represents 1.0. Regardless of the position of the input AC-DC switch, the metering circuitry does not pass frequencies below approximately 5 cycles.

NOTE

The input signal must be properly restricted to a bandwidth comensurate with the instrument's capability. The instrument accepts signals with a bandwidth up to 10 kc. If the attenuation at 20 kc is 20 db or better the instrument will perform well.

MEASURE THE PROBABILITY DENSITY ...
Push the METER SCALE switch to the right to "1.0".
Rotate the OUTPUT-DAMPING-SWEEP TIME switch counter clockwise until a position is reached at which the fluctuations of the PROBABILITY DENSITY meter are sufficiently small.
Rotate the "X" potentiometer to the dial reading corresponding to the dimensionless amplitude at which the probability density is to be read.
Read the probability density on the PROBABILITY DENSITY meter.
Readjust the OUTPUT-DAMPING-SWEEP TIME switch as the "X" dial is changed.

2. Using X-Y Pen Recorder

a. Calibration

CALIBRATE THE PDA ...
The calibration proceeds as in the General Procedure (1 above) with the exception that the METER ZERO is adjusted by observing

the pen position of the recorder. Frequently the recorder is used with an expanded scale so that it has more resolution than the PROBABILITY DENSITY meter. This method of zeroing is therefore necessary. After the PDA has been calibrated the X-Y recorder is calibrated as follows.

CALIBRATE THE X-Y RECORDER X AXIS ...
Push the METER SCALE switch to the SHORT position.
Rotate the FUNCTION switch to the USE position.
Connect the X axis input of the recorder to the X-OUT of the PDA.
Short circuit the Y input to the X-Y recorder.
Rotate the "X" potentiometer fully clockwise. This yields an output voltage of approximately -10 volts.
Adjust the recorder gain to obtain the deflection desired for +5 x RMS.
Rotate the potentiometer back to the zero position (the black "0").
Adjust the recorder parallax for the correct zero position on the recorder paper.
Rotate the potentiometer counter clockwise 5 turns. The output voltage of approximately +10 volts corresponding to -5 x the RMS value is obtained.
Adjust the recorder to correspond to -5 x RMS level.
Repeat the above procedure until the three positions of the "X" dial corresponding to zero, +5 x RMS, and -5 x RMS yield the proper displacement of the recorder pen. For expanding and compressing the "X" scale the gain controls of the X-Y recorder should be used.

CALIBRATE THE X-Y RECORDER Y AXIS ...
Connect the Y axis input of the recorder to the Y-OUT jack of the PDA.

Rotate the FUNCTION switch to the METER ZERO position.
Adjust the recorder parallax until the pen corresponds to the zero position on the chart paper.
Rotate the FUNCTION switch to the CAL position. This produces an output of 2 to 5 volts at the Y-OUT jack. Adjust the recorder Y gain to obtain a deflection on the chart paper which is desired to correspond to a probability density of 0.4. Repeat the above procedure until the METER ZERO and CAL positions of the FUNCTION switch yield deflections on the X-Y recorder corresponding to zero and a probability density of 0.4.

b. Use

SET THE OUTPUT DAMPING ...
Rotate the "X" potentiometer observing the fluctuations in the probability density on the PROBABILITY DENSITY meter.
Select an appropriate OUTPUT-DAMPING-SWEEP TIME position to satisfactorily reduce the fluctuation. The damping time constant and the sweep time have been established to be comensurate in the sense that the blurring effect of sweeping while averaging is minimized. The values of sweep time and damping time-constant can be obtained from the chart at the end of the specifications in section VI, E.

START THE SWEEP ...
Rotate the "X" potentiometer fully clockwise.
Free the friction lock on the potentiometer.
Push the SWEEP switch to the right, to the "+ to -" position.
Lower the pen on the X-Y recorder. A plot of the probability will now be obtained. Alternately, one can start at the opposite end of the "X" axis and sweep in the opposite direction.

3. Using An External Sweep -- Oscilloscope

For definitness the use of the probability density analyzer with an external sweep is discussed in terms of an oscilloscope.

a. Calibration

CALIBRATE THE PDA ...
Calibrate as indicated in 2 above, "Using X-Y Pen Recorder."

CALIBRATE THE OSCILLOSCOPE X AXIS: METHOD A ...
Insert telephone plug from the sweep source into the jack labeled EXT. SWEEP. It is assumed that external controls are available for establishing the excursion of this voltage and the symmetry about zero.
Adjust the external sweep zero level and extent of excursion until the indicating instrument (e.g. an oscilloscope) indicates the same excursion and the same zero location on the "X" axis as is obtained using the "X" potentiometer with the EXT. SWEEP disconnected.

CALIBRATE THE OSCILLOSCOPE X AXIS: METHOD B ...
Rotate the FUNCTION switch to USE position.
Short circuit the INPUT jack.
Connect the Y-OUT to the vertical input of the oscilloscope.
Adjust the oscilloscope to have a vertical deflection sensitivity from 5 to 10 volts full scale.
Connect the oscilloscope sweep voltage through an attenuator and bias source to the EXT. SWEEP jack on the PDA.
Rotate OUTPUT-DAMPING-SWEEP TIME to G.
Initiate the sweep (use approximately 1 sec. sweep). The probability density of zero is infinite for an amplitude

value of zero and non-existent everywhere else. As a result there will be a large y deflection on the oscilloscope when the "X" deflection voltage passes through.
Adjust either the sweep or the "X" deflection on the oscilloscope to place this zero marker at the desired place on the oscilloscope.

Connect -5 volts dc to the INPUT jack on the PDA.
Initiate the sweep. A large deflection will occur on the oscilloscope when the sweep amplitude corresponds to an "X" value of + 5 x RMS.
Adjust the gain on the sweep or the horizontal amplifier of the oscilloscope to obtain the desired deflection.
Invert the DC source to obtain a marker for -5 x RMS and adjust oscilloscope appropriately. Alternate between inputs of +5 volts, -5 volts and a short circuit adjusting the horizontal amplifier of the oscilloscope until the desired X axis deflection and location is obtained.

CALIBRATE THE OSCILLOSCOPE Y AXIS ...
Rotate the FUNCTION switch to the CAL position. Adjust the vertical gain of the oscilloscope to obtain the deflection desired corresponding to a probability density of 0.4.

b. Use

Connect the signal source to the INPUT jack.
Push the METER SCALE switch to the SHORT position.
Rotate the FUNCTION switch to the USE position.
Initiate the sweep from the oscilloscope.
Observe the oscillosope trace.

Adjust the OUTPUT-DAMPING-SWEEP TIME switch to minimize the fluctuation of the display. If the sweep from the oscilloscope is too rapid then in order to obtain sufficient damping the probability density curve may become severly distorted. If the sweep time of the oscilloscope is known, then an appropriate choice of damping time constants can be obtained by extrapolating from the chart given with the specifications in section VI, E.

4. Using a Counter

a. Calibration

CALIBRATE THE PDA ..
Calibrate by the"General Procedure."
Connect the appropriate output (1 or 10 mc counter) with no more than 3 ft. of RG62U cable. Less length is to be preferred especially with the 10 mc OUTPUT.

NOTE

A 10 mc counter is required if accurate measurements are required on waveforms with a spectral content up to 10 kc. The 1 mc counter is only suitable on waveforms with spectral content up to approximately 1 kc.

SET THE COUNTER THRESHOLD ...
Rotate the FUNCTION to the USE position.
Short circuit the INPUT jack.
Set the "X" potentiometer in the 0 position. The counter

should now read continuously. If the counter does not read, switch the METER SCALE switch to the 1.0 position momentarily and observe if an over-scale deflection is obtained. If an over-scale deflection is not obtained on the PROBABILITY DENSITY meter then the unit has not been properly balanced and the initial calibration procedure should be repeated until an over-scale reading is obtained when the METER SCALE switch is placed in the "1.0" position.
Adjust the counter threshold sensitivity. The counter will indicate 1 or 10 mc $\pm$0.1%.

The counter must have a sensitivity comensurate with the 1 volt (nominal) pulses which are obtained from the probability analyzer. It is not wise to adjust the sensitivity of the counter for over-triggering. Most commercial counters have difficulty resolving short random bursts of a few cycles of 10 mc signal since in effect such signals have a higher spectral content than 10 mc. A counter with a direct coupled full 10 mc bandwidth in the amplifier may respond better than one which is ac coupled with large coupling time constants.
Rotate the FUNCTION switch to CAL position.
Adjust the GAIN to obtain a reading on the counter comensurate with the following formula:

$$w(x) = \frac{c}{fT\Delta x}$$

where

$w(x)$ = the probability density
c = the total count
Δx = amplitude interval (this is 0.1)
T = the time interval over which the count is accumulated
f = the frequency designated on the particular output used (that is 1 mc or 10 mc)

The relationship between the probability density and the counts per unit time for the 1 mc and 10 mc outputs should be respectively

$$w(x) = (\text{counts per second}) \times 10^{-5}$$
$$w(x) = (\text{counts per second}) \times 10^{-6}$$

For example, using a 10 mc counter, calibrating at the probability density of 0.4, and using a 1 sec. sampling interval, the GAIN control should be carefully set to yield a count of 400,000 counts. Fluctuations in this count in the third decimal point will be observed.

b. Use

Connect the signal to the INPUT jack.
Adjust the level as described previously.
Rotate the FUNCTION switch to the USE position.
Read the counter output using the formula given above.
The probability density indicated has the argument as indicated on the "X" dial.

In this mode of use the fluctuations are observed by the variations in successive readings on the counter. It will be observed, for example, that for low frequency signals a sample interval of one second may be entirely inadequate and that for sufficiently narrow bandwidth signals a sampling interval of 10 sec. available on the usual counter may not be even adequate. Counters with a scan feature can be used with a stop watch to obtain longer intervals. External timers such as photographic timers may be used when very narrow bandwidth data is to be handled. For automatic data processing a stepped source of direct voltage which also operates the timer can be used on the external sweep. A discussion of the expected uncertainty is presented in section III preceding and examples are given in the Appendix.

5. Using the 2305 Graphic Level Recorder -- Logarithmic Probability

a. Calibration

Calibrate the PDA according to the "General Procedure" (1 above).
Insert potentiometer type ZR0005 in the Graphic Level Recorder type 2305.
Connect the Y-OUT of the PDA to the input of the 2305.
Set the 2305 for DC operation.

ZERO THE OUTPUT ...
Rotate the FUNCTION switch to the METER ZERO position.
Adjust the METER ZERO control so that the stylus on the 2305 is against the stop at -50 db on the scale.

CALIBRATE THE Y AXIS OF THE 2305 ...
Rotate the FUNCTION switch to the CAL position.
Rotate the OUTPUT-DAMPING-SWEEP TIME control to position C.
Adjust the input potentiometer on the 2305, to obtain the desired deflection corresponding to a probability density of 0.4.

CALIBRATE THE X AXIS OF THE 2305 ...
Chose a position of the OUTPUT-DAMPING-SWEEP TIME control in the manner described under the "General Procedure."
Determine the sweep time for this selected position from the chart and the specifications in Part E, section VI.
Select a paper speed on the 2305. The product of the sweep time obtained from the chart and the paper speed set on the 2305 all multiplied by 10 is equal to the number of millimeters of chart paper corresponding to one unit of the RMS.

b. Use

Connect the signal to the INPUT jack on the probability analyzer.
Adjust the input level as previously indicated.
Rotate the FUNCTION switch to the USE position.
Rotate the "X" potentiometer full clockwise.
Lower the stylus.
Start the paper drive motor on the 2305.
Switch the SWEEP switch to the right to the "+ to -" position simultaneously pressing the marker button on the 2305. This indicates the initiating point for the plot.

Recording paper type QP 0102/0302 or QP 1102 are best for this type of recording. By noting the correct ratio of abscissa distance to units of the RMS a template can be laid upon the chart after the recording has been obtained.
Note that the recording is obtained as a logarithm of probability. It should also be noted that probability densities below 10^{-3} are best read with a counter since problems of dc stability occur at these low probability densities.

6. Obtaining Exceedances

a. Calibration

Calibrate as indicated under "General Procedure" (1 above).
Connect the input of a counter to the PULSE OUTPUT jack on the rear of the PDA. The connecting cable should be as short

as possible and be of type RG62U.
Set the counter at maximum sensitivity.

b. Use

Rotate the "X" potentiometer to the level at which the exceedances are to be measured.
Connect the input signal to the INPUT jack and adjust the input level as previously described.
Rotate the FUNCTION switch to USE position. The counter will now indicate a count for each time the signal is in the region about the "X" level. The amplitude interval is sufficiently small that the probability of the waveform going into this interval and not passing through it is extremely small. Therefore, the indication on the counter, divided by 2 is the number of exceedances. If the counter is in a "counts per unit time" mode then exceedances per unit time are obtained. When the "X" dial is set at zero the counts per unit time divided by 2 is the "average frequency" of the random signal.

7. Obtaining Distribution of Peaks (Powell Method)

Two types of the distribution of the peaks are in common use. The one method we denote the Powell Method[15] and the second method we denote the Envelope Method. In this section the procedure for obtaining the peaks by the Powell Method is illustrated.

In the Powell Method one is actually considering the distribution of the exceedances. The procedure is quite simple. It is desired to take the ratio of exceedances above the "set-in" level to the exceedances above zero. The exceedances above zero are of course 1/2 the zero crossings. With the exception

of the comparatively infrequent occurence of maxima and minima which do not go through the zero crossing it can be seen that the Powell Method tends to yield a normalized probability density function. It is important to recognize however that there is nothing in the measurement technique to guarantee that the distribution thereby obtained will be normalized. A case of interest is a sum of a very low frequency large amplitude random signal and a high frequency band of random noise. In such a case, the distribution of peaks by the Powell Method will not be normalized.

The use of the Powell Method is primarily in fatigue studies. The numerical value obtained by this technique when multiplied by an assumed mean frequency of the excitation yields a number which is approximately equal to the number of exceedances of a specified level per unit time. In this sense the Powell Method is a sort of dimensionless method of obtaining exceedances as discussed in procedure 6 above.

a. Calibration

The calibration is performed as indicated in "Obtaining Exceedances" (6 above).

b. Use

OBTAIN THE DISTRIBUTION BY CALCULATIONS WITH COUNTER ...
Proceed as indicated in "Obtaining Exceedances" (6 above).
Note the number of exceedances for each setting of the "X" potentiometer.
Divide each of these readings by the number of exceedances that occur when the "X" potentiometer is set at 0.

Plot these results versus the dimensionless amplitude. This produces the distribution of the peaks.

OBTAIN DISTRIBUTION OF THE PEAKS AUTOMATICALLY ...

Connect the pulse output of the PDA to a counting rate indicator such as the HP Model 500B. The counting rate meter then indicates directly the average exceedances per unit time.
Connect the output of the rate meter to the Y axis of an X-Y recorder.
Connect the X axis to the X-OUT jack of the PDA.
Calibrate the X axis as indicated under the procedure for using an X-Y pen recorder 2 above.
Rotate the "X" potentiometer to the 0 position.
Adjust the rate meter or the X-Y plotter Y axis gain to obtain full scale deflection.
Rotate the OUTPUT-DAMPING-SWEEP TIME control fully counter clockwise.
Rotate the "X" potentiometer full clockwise.
Push the SWEEP switch to the right to the "+ to -" position.
The X-Y recorder will now make a plot of the probability of the peaks as defined by Powell.

8. Obtaining Distribution of the Peaks (Envelope Method)

The Envelope Method for obtaining the distribution of the peaks refers to the signal that would be obtained by applying an input signal to a peak detector. The probability of the peaks as measured this way is only significant for essentially narrow band processes, such as the output of the audio-frequency spectrometer type 2112 or the frequency analyzer type 2107.

a. Calibration

Calibrate the PDA as indicated in "Using an X-Y Pen Recorder" (2 above).

b. Use

GENERATE THE ENVELOPE OF THE INPUT SIGNAL ...
Connect the "high" side of the input signal to a diode such as type 1N58A.
Connect the diode to a load resistor of no less than 10 x the source impedance.
Pass the signal across the load resistor through a filter-amplifier. This should be a DC pass unity gain system with a sharp cut-off frequency at a frequency 1 octave greater than half the bandwidth of the input signal and with a rejection of at least 40 db at a frequency 1 octave below the mean frequency of the input signal.
Connect the output of the filter to the INPUT jack of the PDA.

ANALYZE THE SIGNAL ...
Use the PDA as indicated in "Using the X-Y Pen Recorder" (2 above). The data obtained will be the probability density of the peaks of one polarity of the input corresponding to the polarity with which the diode was inserted. The probability density of the peaks of the opposite polarity is obtained by inverting the diode.

9. Obtaining Distributions of Functions of the Input

It is sometimes desirable to obtain probability densities of some function of the input. For example the previous paragraphs on

"Obtaining the Distribution of the Peaks" (8 above) considered a function of the input. The method outlined in 8 is that generally used.

As an example of a particular function which is of interest consider the probability density of the energy. In this case the procedure illustrated in 8 above is followed. However, instead of the diode and filter circuit as indicated there, a squaring circuit with unity gain is placed between the signal and the input to the PDA. This squaring circuit must have essentially no averaging so it reproduces the instantaneous value of the square of the input signal. One thereby obtains the probability density of the energy.

NOTE

Non-linear operations on the input signals in general introduce signal components which extend the original spectrum. The performance range of the PDA must be considered if such operations are performed. For example, in the case of a squaring circuit the actual input spectrum must be restricted to 5 kc because of the frequency doubling effect of the squaring circuit.

10. Obtaining Joint Probability Density

A detailed discussion of these techniques is beyond the scope of this report. It is important to recognize however that without further external equipment joint probability can be obtained with

two probability density analyzers and a counter having a coincidence input. The output of one probability analyzer is supplied to "input A" and the PULSE OUTPUT of the other probability density analyzer is supplied to "input B". The counter is then set in the mode in which it counts "a" while "B" is on. Joint probability density is then obtained.

The most useful form for obtaining the joint probability density is in the form of the conditional probability. The probability density analyzer whose PULSE OUTPUT is being used is set to a fixed value. The other probability density analyzer is moved through its values in the usual manner. This then becomes a conditional probability density of the form: given B, what is the probability density of A. This can be plotted for a series of set-in values of "X" in probability density analyzer B. Higher order statistics can be obtained in a similar manner. For several conditional probabilities it is necessary to combine the PULSE OUTPUT of the several conditional probability density analyzers through summing resistors and to set the threshold level on the counter so that "B" trigger will only open when all the pulses occur at the same time. The signal is assumed, of course, to be supplied in parallel to the INPUT jack of all the probability analyzers. Since the input impedance of each PDA is 10,000 ohms a low source impedance is required.

Time statistics of higher orders can only be obtained if an external time delay is available.

11. Obtaining Amplitude Density

The distinction between amplitude density and probability density concerns primarily the normalization procedure. An amplitude density curve is the measure of the relative time the signal spends

at a given amplitude. This satisfies the definition of probability density if the proportionality is such that the relative times integrated over the amplitudes is equal to unity. All the operational procedures and types of measurement indicated so far have been in terms of dimensionless amplitude, that is the ratio of the instantaneous amplitude to the root mean square value of the signal. Considered here are procedures for obtaining measurements of the distribution of amplitudes in signals where it is not feasible to employ a normalization procedure. Such signals are typically certain speech waveforms, gun and cannon noise, transients in reverberant spaces, and certain types of responses to a stimulus in electroencephelographic data. This method can also be used in handling some vibration data.

For definiteness we use as an example say, the transient acceleration response of a structure. If this response lasts for a few hundred milliseconds or even for several seconds is difficult to specify the root mean square value. Furthermore if the transient is a result of the superposition of many decay constants it may be difficult to decide as to when the transient response in the sample should be terminated for analysis. The plot we wish to obtain is some measure of the relative time that the acceleration is in the region of certain amplitudes as a function of these amplitudes.

a. Calibration
Calibrate the PDA as indicated in the procedure "Using an X-Y Recorder" (2 above).

ESTABLISH THE INPUT LEVEL ...
Rotate the "X" potentiometer fully clockwise.
Rotate the FUNCTION switch to USE position.
Push the METER SCALE switch to the 1.0 position.

Connect the signal to be analyzed to the INPUT jack.
Observe the PROBABILITY DENSITY meter.
Rotate the input potentiometer until some response is obtained on the PROBABILITY DENSITY meter. For more precise adjustment the threshold response can be detected on a counter that is connected to the 1 mc or 10 mc output. In that case one rotates the input potentiometer until at least one count is obtained on the counter.
Rotate the input potentiometer counter clockwise until the response just disappears.
Rotate the "X" potentiometer fully counter clockwise.
Observe the PROBABILITY DENSITY meter or counter to determine if there is some response. If there is some response rotate the input potentiometer in a counter clockwise direction until the response vanishes. The transient has now been scaled so that the maximum peak of the transient is no greater than full scale range of the instrument in either polarity.
Disconnect the signal from the INPUT jack.
Connect a calibrated source having a sinusoidal waveform.
Adjust the source amplitude until a response is just obtained on the indicating instrument.
Note the amplitude of the source. The peak value indicated by the source now corresponds to FULL SCALE of the "X" dial (the "X" dial was left in the fully counter clockwise position during this last operation).

b. Use

READ THE AMPLITUDE DENSITY ...
Reconnect the signal to be analyzed to the INPUT jack.
Set the "X" potentiometer to the desired amplitude using the

calibration obtained as previously indicated.
Intiiate the transient. If only a single transient is available but on a magnetic tape loop, a scanning technique can be used. In general, however, the transient must be initiated for each measurement.

CHOOSE THE TRANSIENT INTERVAL ...
It is not necessary to actually truncate the transient if it is desired to remove contributions to the probability density beyond which the transient is essentially zero. After the plot of the amplitude density is obtained, the portion of the plot corresponding to an interval around zero will have a very large value if a large amount of "dead" space has been included. One merely replots the data eliminating this contribution at the zero level. The curve is still an amplitude density. In order to normalize it, to obtain a probability density, graphical integration is required.

In those cases when it is desired to obtain the probability density of a transient and it is followed not by "dead" space but by additional signals, it is advisable to gate the signal. A comparatively simple way of accomplishing the equivalent of gating a signal can be used when the output device is a counter. The transient is used to activate the input gate to the counter. In this way counts which are obtained from the undesired parts of the transient are not contributed. The advantage lies in the fact that it is easier to gate pulses than to gate the input signal without distortion.

12. Obtaining Unnormalized Exceedances

a. Calibration

Calibrate according to the operational procedure, "Obtaining Amplitude Density" (11 above).

b. Use

Connect a counter to the PULSE OUTPUT. Plot the readings obtained versus the "X" dial reading corrected according to the calibration.

13. Obtaining the Probability Distribution

The probability distribution function, P(x) (also called the cumulative probability) is a measure of the probability of the random variable being less than a specified value. Another function frequently required is the probability of a random variable being greater than some value; this is simply [1 - P(x)]. The following properties of P(x) are of interest. (1) $P(x) \leq 1$. (2) The probability density is related to the probability distribution by $P(x)=\int_{-\infty}^{x} w(\xi)d\xi$. (3) In a practical measurement direct evaluation of P(x) tends to lose information whereas P(x) can always be recovered from a direct measurement of w(x), the probability density.

The simplest method of obtaining P(x) is to graphically integrate the probability density from $-\infty$ (-5σ) up to a value, x. When this is plotted versus x one has the probability distribution P(x). An automatic method using a counter is indicated on the following page.

a. Calibration

Calibrate the PDA as indicated in the procedure "Using a Counter" (#4, Page 48).

SET UP EXTERNAL DRIVE AND INTEGRATION APPARATUS ...

Set the counter in "count" or "totalize" or "accumulate" mode.
Connect a printer to the counter.
Connect a voltage divider across the stationary contacts of a 100 position stepping switch so that voltages from -10 to +10 volts in 0.2 volt increments are available.
Connect the moving contact of the switch via a telephone plug to EXT. DRIVE.
Rotation of this switch will step the set-in level discontinuously through contiguous intervals of 0.1σ.
Connect a 1 sec interval timer so as to deliver an actuating pulse to the rotary switch and a print command to the printer.

b. Use

Connect the signal to the INPUT jack.
Adjust the level as described previously.
Rotate the stepping switch to the -5σ (+10v) position.
Rotate the FUNCTION switch to USE position.
Reset the counter.
Start the timer.

NOTE
The counter is used as totalizer do not reset until end of run. (Step switch through 100 pos.)

Read the printer listing using formulas of Page 50 but multiplying by 10^{-1}. The printer lists P(x) vs. x. There are 100 readings for x from -5 to +5 in 0.1 increments. The last reading should be 1.0 if the signal is contained in -5 to +5 times the rms.

B. Detailed Theory

Figure 15 is a circuit diagram of the PDA. The function indicated by the block 1 of Fig. 2, describing the waveform up to illustration d of Fig. 3 and indicated as "input circuits" in Fig. 4 are performed by the circuitry associated with vacuum tubes V 1, V 2, and V 3. The signal is presented to the input. It can be channeled via a 2 μf coupling capacitor or by a direct path to a 10,000 ohm 10 turn potentiometer. The selection of a direct path or the capacitor path is the function of the ac/dc switch S 1. The 10 k ten turn potentiometer is the input potentiometer (DIVIDE BY).

In the USE position the output of the 10 turn potentiometer is channeled to the grid pin 2 of V 1, a cathode follower. The output of this cathode follower feeds two circuits.

One circuit is the amplifier V 3. V 3 is an amplifier for the thermocouple which operates the input RMS meter. The RMS meter switch is a shunt across the thermocouple to protect it from overloading. In addition, switch wafer 2B of the FUNCTION switch shorts out the thermocouple for all functions except CAL and USE. Rear panel adjustment C is available for calibration of the thermocouple meter. This is the 20 k potentiometer in the cathode circuit V 3.

Return now to the signal at the first cathode of V 1. Note that this is supplied via a summing network to the grid, pin 2 of V 2. The other input to the summing network is a reference voltage which is obtained from the "X" potentiometer or via the external drive jack when an external sweep is used. Calibration of this summing network is obtained through adjustment D, a 100 k potentiometer from the cathode, pin 3 of V 1. V 2 also comprises a pair of cathode followers. The input to the following circuitry is the voltage across pins 3 and 8, the cathodes of V 2.

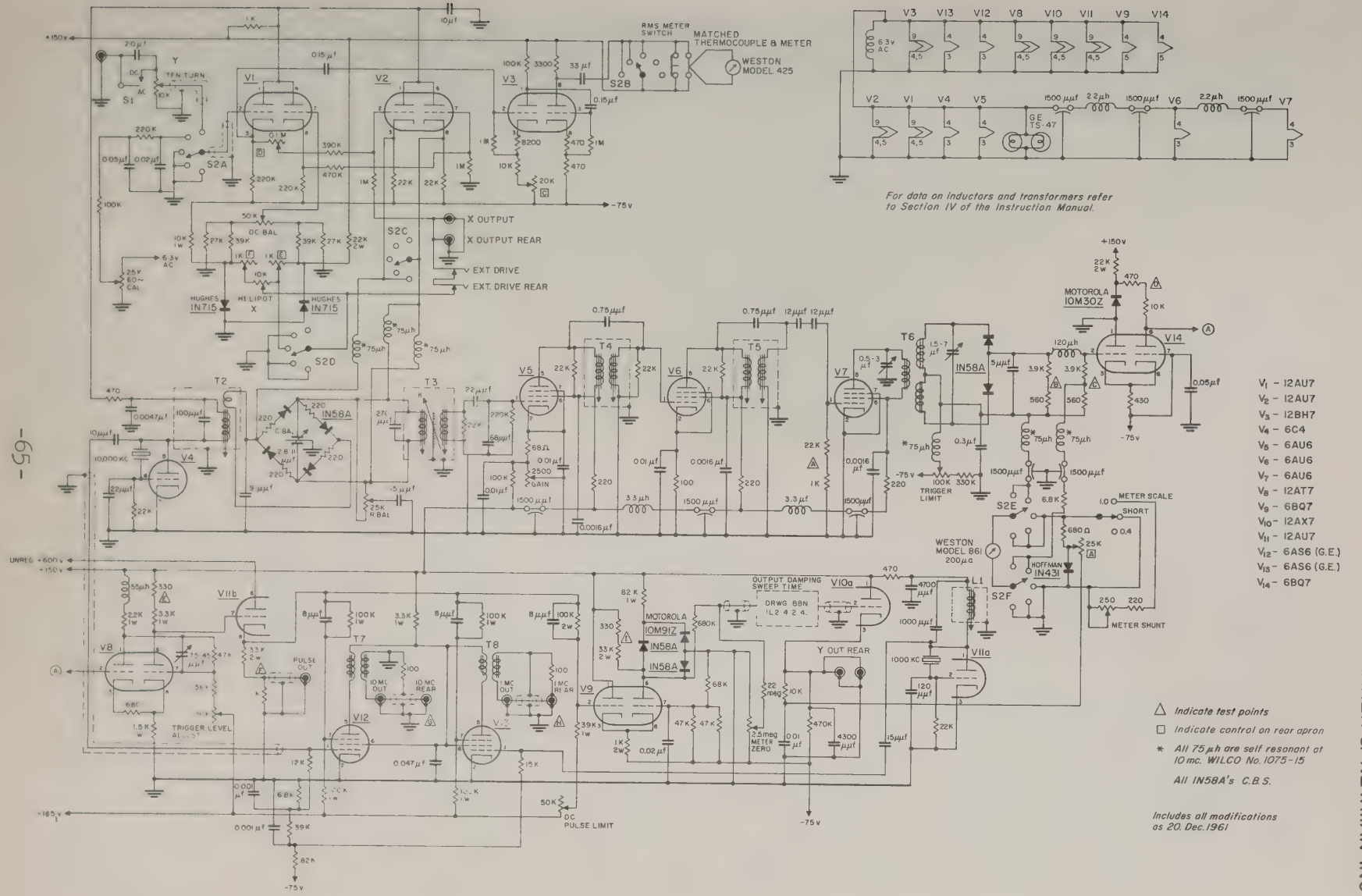


FIG. 15 CIRCUIT DIAGRAM - PROBABILITY DENSITY ANALYZER MODEL B.

BBN I.L. 2.4.2. 21.3

To determine the signals available at pin 8 of V 2 note that the cathode follower is supplied from the corresponding cathode follower in V 1. Pin 7 is supplied from the balance potentiometer, DC, available behind the drop-out panel. Through these adjustments a null voltage is obtained across pins 3 and 8 of V 2 when the input is zero and when the "X" potentiometer is set in the 0 position.

To establish the proper range of the "X" voltage, adjustments F and E have been provided. These adjust the end voltages on the ten turn "X" potentiometer. These voltages are obtained in a stable way by the use of Zener diodes as indicated.

In the CAL position of the FUNCTION switch, S2A supplies a 60 cycle voltage to the input of V 1. This is obtained from the filament winding of the power transformer and adjusted by the control behind the drop-out panel labeled CAL. This 60 cycle is filtered through a double RC low pass filter to remove harmonic components. In this position of the FUNCTION switch, S2D short circuits the slider of the "X" potentiometer to ground insuring that the measured probability will correspond to the probability density of a 60 cycle signal at the zero crossing.

In the third counter clockwise position of the FUNCTION switch, S2D establishes a reference voltage corresponding to 5 x RMS. S2A grounds the input. The result is a probability density of zero. The PROBABILITY DENSITY meter can be zeroed by the METER ZERO adjustment.

Returning now to the discussion of the signal obtained across pins 3 and 8 of V 2, we note that this provides the audio input to a balanced bridge modulator. The signal is supplied through

set of chokes which prevent the radio frequency from appearing in the audio circuits. The supply for the bridge is a 10 mc oscillator comprised of V 4 and associated circuitry. The oscillator is crystal controlled. The balancing of the bridge requires a capacitance balance and a resistance balance as indicated. These are the R and C controls located behind the front panel. The output of the bridge is coupled into an IF amplifier through a special coupling transformer with a Faraday shield.

The IF amplifier consists of vacuum tube V 5, V 6 and V 7. V 5 and V 6 are IF amplifiers and V 7 is a limiter. The bandwidth required to preserve the integrity of the detail of the waveform at the point of the intercept with a reference voltage is larger than the bandwidth of the input signal. The half power bandwidth of the IF amplifiers without the limiter has been established between 800 kc and 1 mc. Minimum phase asymmetry is maintained throughout. The potentiometer in the cathode circuit of V 5 is the GAIN control behind the front drop-out panel. The bandwidth of the limiter V 7 and its special transformer is established at between 3 and 4 mc. The output of the limiter is detected by a full wave rectifier. The signal appearing at the output of this full wave rectifier corresponds to the waveforms in illustration g of Fig. 3.

To explain how the waveform indicated in illustration d of Fig. 3 was converted to the waveform g of Fig. 3, consider Fig. 16. The waveform d is supplied to the bridge. The bridge, being a balanced modulator, produces the waveform D' indicated in Fig. 16. Amplifying a small segment of this wave as indicated in illustration e of Fig. 3 produces illustration E' of Fig. 16 (note the actual ratio between the rates of change of the waveform and the fine structure

FIG. 16 TYPICAL WAVEFORMS IN THE ARGUIMBAU CIRCUIT

of the modulation would be much greater, but would not show up well in the illustration). The output of the limiter, vacuum tube V 7 produces the waveform f' of Fig. 16. This corresponds to the wave f of Fig. 3.

This wave is full-wave rectified. A small amount of filtering is accomplished by the π section filter comprised of the 5 μμf capacitors and the 120 microhenry coil. The waveform of g is produced. Only the details of this waveform are given. Values much larger than those shown are also limited by achieving cut-off with the negative swing at grid pin 1 at V 7.

As indicated in waveform g of Fig. 3 the gain applied to the differential signal establishes the effective amplitude interval. For this reason the GAIN control in the cathode of V 5 is the control for the amplitude interval.

Functions through block 3 of Fig. 2 are performed in the circuitry described thus far. The waveform generation indicated by block 4 of Fig. 2 and by the waveform h of Fig. 3 is accomplished through the circuitry associated with vacuum tubes V 14, V 8 and V 11 B and V 9. Photographs of some of these waveforms are presented in the section on alignment.

The function of V 14 and associated circuitry is to select the mid-portion of the waveform exemplified by g of Fig. 3. V 14 is a cathode coupled limiter. The use of the Zener diode for the plate supply establishes the mean operating potential at the output plate to be commensurate with the firing potential of the Schmidt Trigger V 8. To establish the proper operating potential for grid pin 2 of V 14 an internal adjustment designated "trigger limit" is provided.

The Schmidt Trigger V 8 is a conventional Schmidt Trigger with a peaking circuit in the first plate and hysteresis minimizing resistor in the first cathode. The proper firing potential of the Schmidt Trigger is established by the potentiometer labeled "trigger level adjust".

The output pulse of the Schmidt Trigger is supplied to grid pin 7 of V 11 B, a cathode follower. This pulse is available through a divider network at the PULSE-OUT jack. It is supplied to three circuits, those associated with V 12, V 13, and V 9.

The function of vacuum tube V 9 is to establish a large, constant amplitude pulse which is suitable for RC integration to obtain an analog output. V 9 is also a cathode coupled limiter. An internal adjustment, "dc pulse limit" is available for insuring that the first triode of V 9 will be cut off when no pulse is present. The second triode is then in conduction. However, the diode 1N58A clamps the plate of the second triode at approximately ground potential during conduction.

When the pulse occurs the first tube conducts heavily cutting off the second tube. However, another diode in conjunction with a Zener reference diode establishes a maximum amplitude to the pulse. These pulses are supplied via a 680 k resistor to a set of RC π networks. These time constants are selected by the OUTPUT-DAMPING-SWEEP TIME switch.

A small voltage is available through a 22 megohm summing resistor and a 2.5 megohm potentiometer to offset the small potential drop across the diode which approximately clamps this waveform to ground. This potentiometer is the front panel control METER ZERO.

The RC network performs the averaging indicated in block 5 of Fig. 2. The voltage from the RC network is supplied to a cathode follower which comprises an output dc impedance converter. The voltage is available at the Y-OUT jack. This direct voltage is also supplied to the PROBABILITY DENSITY meter. To obtain two scales on the PROBABILITY DENSITY meter a shunt is provided. Internal adjustment is used to establish the proper scale ratios. The METER SCALE switch selects the shunt for the meter or short circuits the meter. The calibration of the meter is obtained through the adjustment A, a 25 kilo ohm potentiometer. To protect the meter from overload a Zener diode is connected as indicated.

Switch wafers S2E and S2F are on the FUNCTION switch. They place the meter across the output of the full wave rectifier when the DC BAL and RC BAL are to be adjusted. In this position the meter registers a current proportional to the unbalance voltage from the bridge.

An equivalent averaging is accomplished by gating pulses as indicated in waveform i of Fig. 3. This is accomplished with the circuitry associated with vacuum tube V 12 and V 13. These are identical gating circuits. The pulse available through the cathode of V 11 B is supplied to the suppressor grid, pin 7. Grid pin 1 of V 12 receives a signal properly biased from the 10 mc oscillator. The signal on grid pin 1 of V 13 is supplied from the independent 1 mc oscillator. This is also crystal controlled. It is comprised of V 11 A and the associated circuitry. Aperiodic transformers are used for impedance conversion to supply the 10 and 1 mc counter outputs. These transformers are made to be matched to 93 ohm cable (RG62U).

The power supply for the probability density analyzer is indicated in Fig. 17. It is in most respects conventional. A regulated 150 volt supply is available. The unregulated 600 volt supply is used for V 11 B in the PDA. A -75 V Zener diode regulated supply is included and a -165 V Zener diode regulated supply. These latter supplies obtain their alternating voltage by capacitive coupling to the power transformer. The capacitor filter network on the -165 V bias supply is connected in such a way as to bleed the AC current from passing through the -75 V supply.

C. Alignment

Alignment of the PDA is accomplished by utilizing standard laboratory techniques.

1. Calibrate the matched thermocouple and INPUT meter.
Apply a 1 V RMS signal. Rotate the input potentiometer full clockwise. Adjust potentiometer C for a reading at the fiducial mark on the upper part of the INPUT meter scale. Adjust potentiometers E and F to obtain ± 5 V at the ends of the 10,000 Ω 10 turn "X" potentiometer.

2. Align the counter dial on the "X" potentiometer.
The 0 indication on the dial must correspond to a zero potential between ground and the slider of the potentiometer.

3. Adjust the summing resistors.
Rotate the input potentiometer full clockwise. Adjust potentiometer D such that precision DC inputs corresponding to settings of the "X" potentiometer produce a null at the junction of the 390K and 1 megohm resistors to grid pin 2 of V 2.

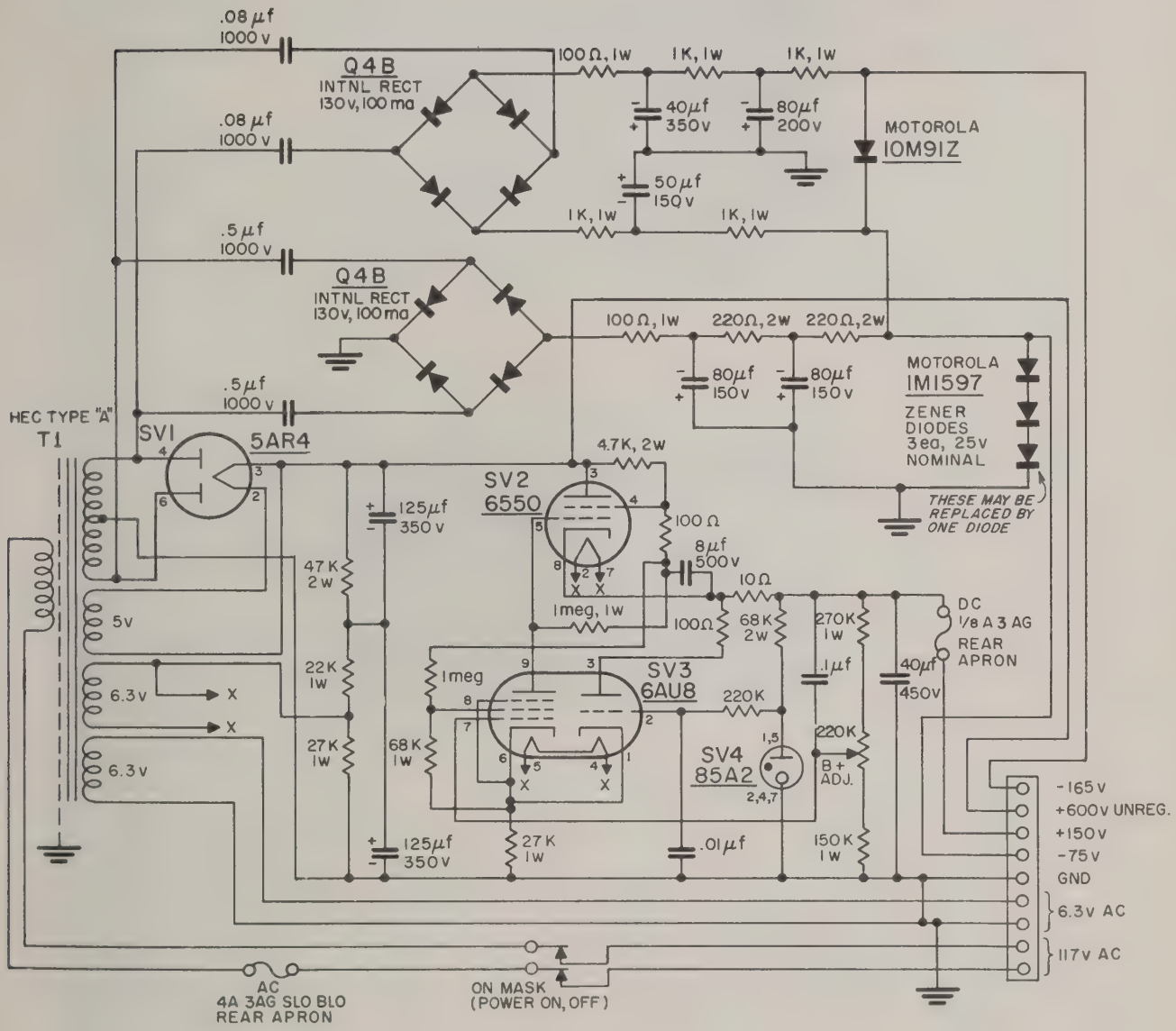


FIG. 17 CIRCUIT DIAGRAM – POWER SUPPLY

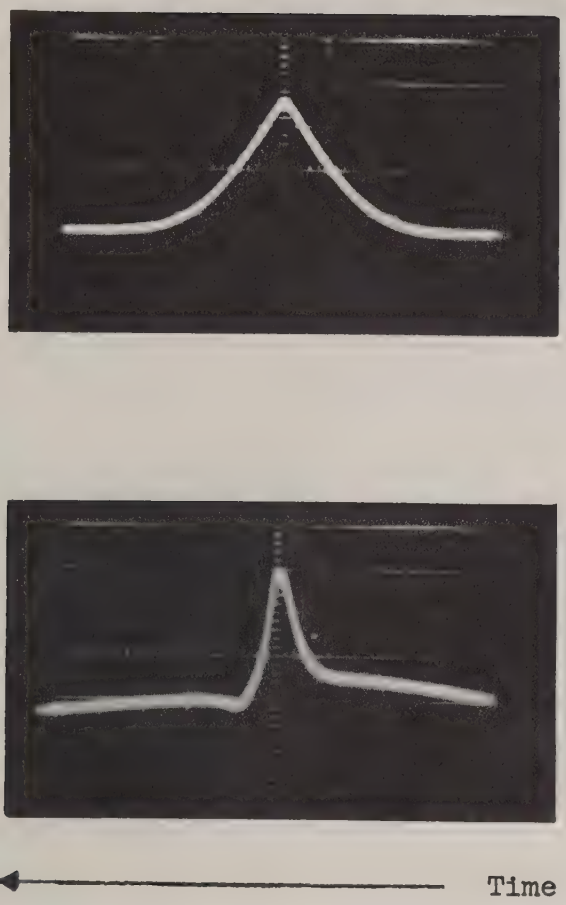
4. Adjust the ten megacycle oscillator.
Adjust transformer T2 so that the 10 mc oscillator oscillates at maximum amplitude and is stable with respect to small variations in the inductance.

5. Align the IF strip.
Alignment of the IF strip is accomplished with a sweep generator and marker. The limiter transformer is tuned by the variable capacitors across the secondary and primary. Symmetric passband ripple of up to 1 db due to overcoupling may be obtained. The passband is 3 to 4 mc about 10 mc.

The alignment of the remaining stages is then accomplished by tuning the T5, T4, and T3 in that order. The coupling of T3 is adjustable. The bandwidth of the IF strip should be between 800 kc and 1 mc. A convenient way to determine if the IF gain-bandwidth is adequate is to observe the signal at pin 2, V 14 with V 14 removed. Fig. 18 was obtained at this point. With an input signal of a 1 kc sine wave and the "X" potentiometer set at 0, the waveform A is obtained. An input of 10 kc at a level of 11 db above 1 volt yields the waveform of B at this point.

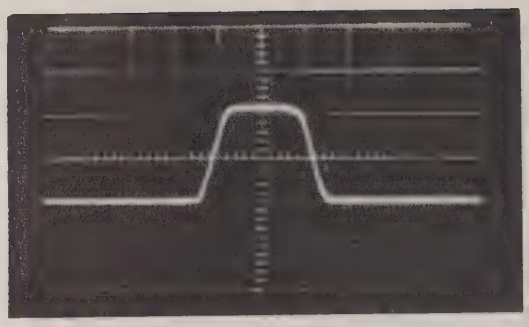
6. Adjust "trigger limit" potentiometer.
This is adjusted until the correct portion of the waveform (Fig. 18) is selected by the cathode coupled limiter V 14. Fig. 19 indicates a properly adjusted "trigger limit." The signal s measured at test point D on the plate circuit of V 14. In A of Fig. 19 the input was a 1 kc signal at 1 volt. For B the input was a 10 kc signal 11 db above 1 volt.

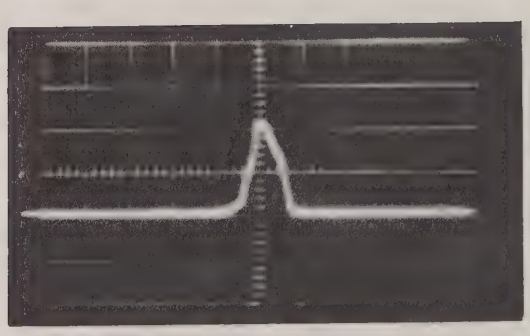
7. Set trigger level.
Adjust the commutating capacitor coupling the first plate to the second grid of V 8 at approximately 50% rotation. Set the "trigger



A ...	Location:	pin 2 V 14	
	Calibration:	vertical	5 volts/cm
		horizontal	5 μs/cm
B ...	Location:	pin 2 V 14	
	Calibration:	vertical	5 volts/cm
		horizontal	0.5 μs/cm

Figure 18: Waveforms (see text for input and circuit conditions)





A ...	Location:	test point D	
	Calibration:	vertical	0.5 volts/cm
		horizontal	5 μs/cm
B ...	Location:	test point D	
	Calibration:	vertical	0.5 volts/cm
		horizontal	0.5 μs/cm

Figure 19: Waveforms (see text for input and circuit conditions)

level adjust" by referring to Fig. 19. Note the loading distortion in the waveform. The location of these points on the waveform will change with adjustment of "trigger level adjust." Fig. 19 indicates proper adjustment.

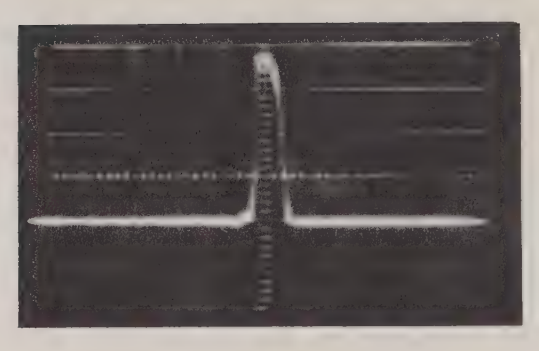
8. Calibrate the GAIN.
Refer to the normal calibration procedure. Adjust the RC BAL and the DC BAL as indicated. Examine the waveforms at test point E. Rotate GAIN to obtain time durations as indicated in Fig. 20. The waveform at A is obtained with a 1 kc 1 volt input signal and the waveform illustrated in B is obtained with an input of 10 kc + 11 db RE 1 volt.

9. Adjust overshoot.
Readjust the 7 1/2 mmf commutating capacitor between the first plate and the second grid of V 8 to obtain waveforms such as illustrated in Fig. 20. It is most convenient to adjust with a 1 kc signal so that the compensation is critical as illustrated.

10. Adjust the "dc pulse limit."
The output of V 9 at the plate, pin 6 should be zero based and peak limited. The waveform desired is indicated in Fig. 21. This waveform was obtained with a 1 kc at 1 volt.

Measurement of this waveform with a higher frequency and/or higher amplitude input will display the effects of capacitive loading. A test point exists at I. If this test point is to be used, place a capacitor at the supply side of the 330 ohm resistor to ground. This must be physically close to the resistor otherwise the inductance of the leads, etc., causes the waveform to be distorted.

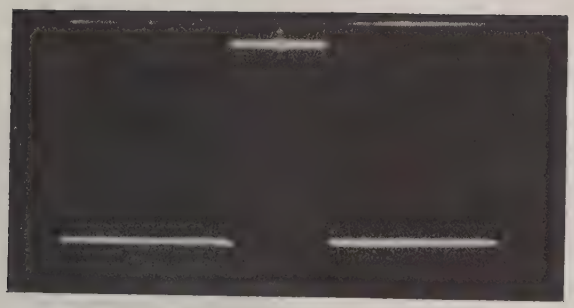
A



B

A ...	Location:	test point E	
	Calibration:	vertical	1 volt/cm
		horizontal	5 μs/cm
B ...	Location:	test point E	
	Calibration:	vertical	1 volt/cm
		horizontal	0.5 μs/ cm

Figure 20: Waveforms (see text for input and circuit conditions)



Time

Location:	pin 6 V 9	
Calibration:	vertical	20 volts/cm
	horizontal	5 μs/cm

Figure 21: Waveforms (see text for input and circuit conditions)

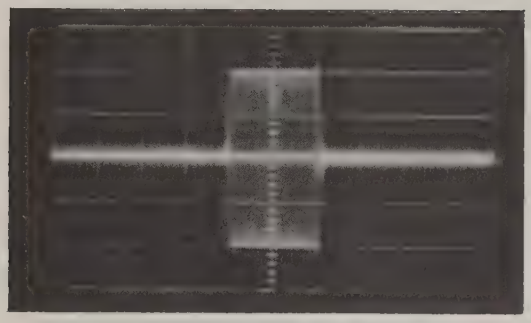
To verify that the "dc pulse limit" has been properly adjusted, the normal calibration procedure for adjusting METER ZERO can be performed. If the METER ZERO adjustment cannot properly zero the meter, the adjustment of the "dc pulse limit" may be in error.

11. Adjust the 1 mc oscillator, and check gates.
Adjust the slug in inductor L 1 in the 1 mc oscillator (VII A) until the oscillator is oscillating at maximum amplitude. To guarantee stability verify that oscillation is maintained if the slug is rotated in either direction from maximum amplitude.

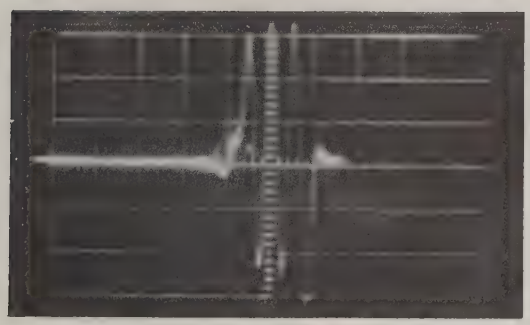
Verify proper operation of the gate V 12 and V 13. The signals are examined at test points G and H, the 10 mc and 1 mc counter outputs. The waveform obtained at the 10 mc output is illustrated in Fig. 22. The input signals for A and B of Fig. 22 are as previously indicated. In Fig. 23 the output from the 1 mc jack is indicated. The waveform of B in Fig. 23 reflects a coincidence of the 1 mc pulse and the gating pulse. It does not imply that the 1 mc output should be used at 10 kc.

12. Adjust PROBABILITY DENSITY meter.
Use the calibration procedure described under section VI, A, 4, "Use with a Counter". After the counter reading has been established at the probability density of 0.4, adjust potentiometer A to obtain this reading on the meter. The METER SCALE switch is used to alternate between the 0.4 and 1.0 scale. The METER SHUNT is adjusted to obtain the same reading.

This completes the alignment procedure.



A

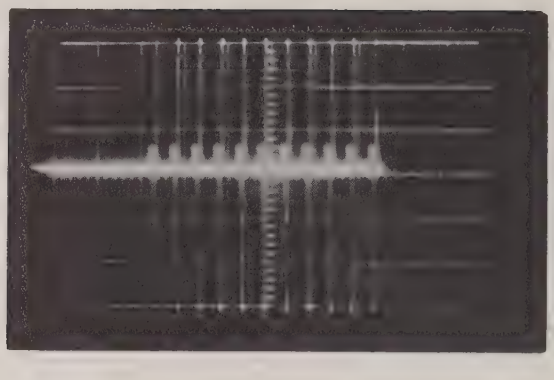


B

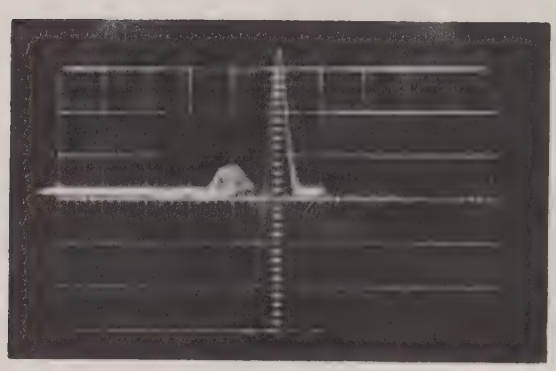
Time

A ...	Location:	test point G	
	Calibration:	vertical	1 volt/cm
		horizontal	5 μs/cm
B ...	Location:	test point G	
	Calibration:	vertical	1 volt/cm
		horizontal	0.2 μs/cm

Figure 22: Waveforms (see text for input and circuit conditions)



A



B

A ...	Location:	test point H	
	Calibration:	vertical	1 volt/cm
		horizontal	2 μs/cm
B ...	Location:	test point H	
	Calibration:	vertical	1 volt/cm
		horizontal	0.2 μs/cm

Figure 23: Waveforms (see text for input and circuit conditions)

D. Performance Data

Several of the illustrations that were presented in section V on the use of the PDA were taken with the model 160. In the following are presented some additional curves which were taken expressly to indicate the performance of the model 160 PDA. An Electronic Associates X-Y recorder was used. The sine wave signals were obtained from a Hewlett Packard audio frequency oscillator. The random noise was taken from a gas tube type noise generator made by Grason-Stadler. In Fig. 24 is illustrated the measured probability density of a 100 cycle sine wave. Fig. 25 is of a 1,000 cycle sine wave. Fig. 26 is of a 10,000 cycle sine wave. Fig. 27 is of the random noise obtained from the Grason-Stadler noise generator when the spectrum switch is placed in the SPEECH position. This is noise which has a spectrum similar to the usual spectrum of speech. It is entirely within the frequency range of the model 160 PDA. These groups of curves show performance in the normal range of operation of the instrument.

To give some indication of the tolerance on performance of the instrument, the performance beyond its specified operating range is indicated in Figures 28 through 30. The curves, Figures 28 and 29 are the probability density of a sine wave of 20 kc and of 30 kc respectively. Fig. 30 is the probability density of random noise obtained from the Grason-Stadler noise generator when the spectrum switch is in the "high" position, yielding noise with a spectrum in excess of 20 kc.

The sine wave curves are better indicators of the instrument's performance since there is no easy way to cross check the performance of the instrument on random noise. In both Figures 27 and 30 one observes an asymmetry in the probability density which

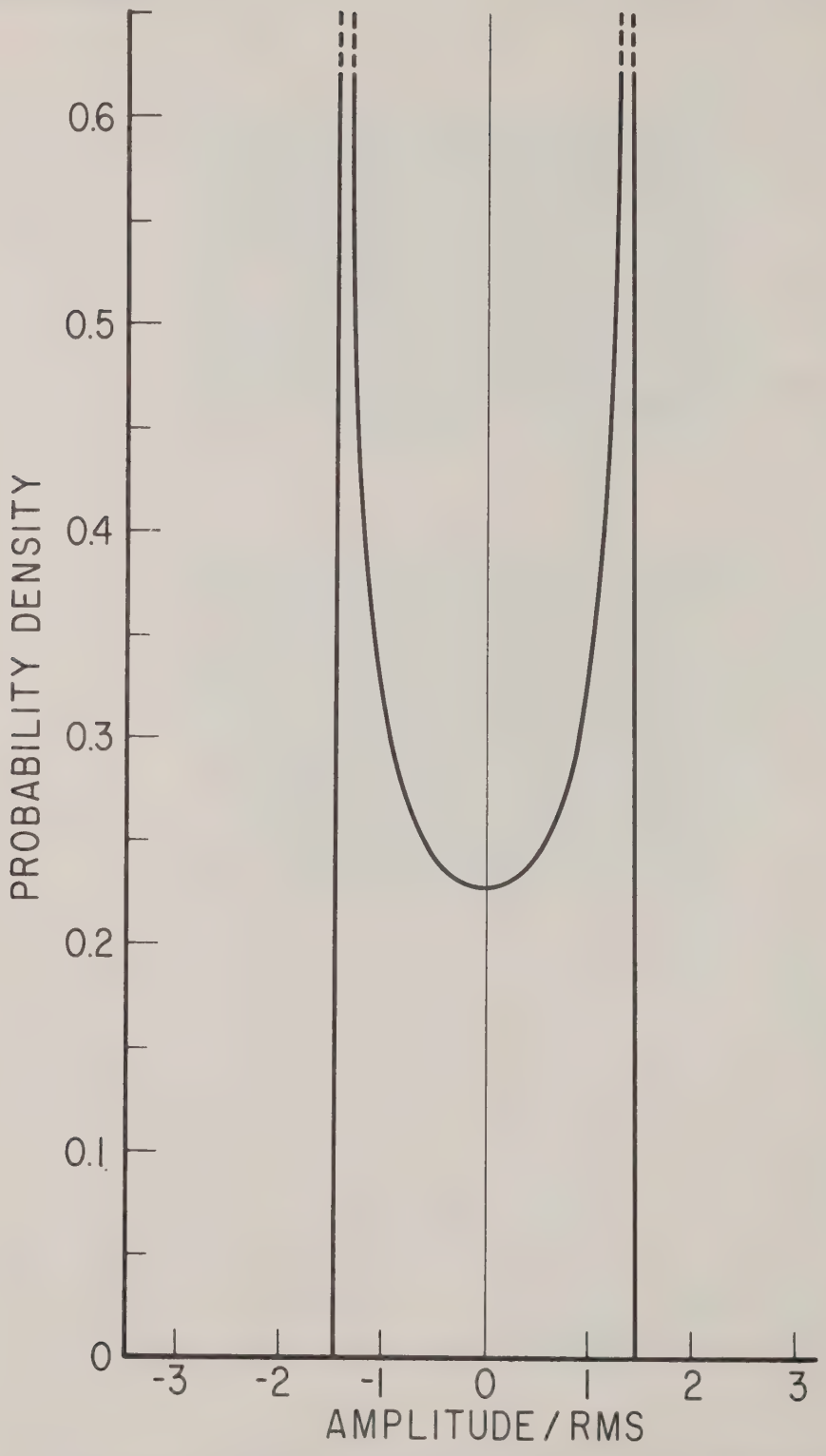


FIG. 24 PROBABILITY DENSITY OF 100 CPS SINE WAVE MEASURED BY MODEL 160B

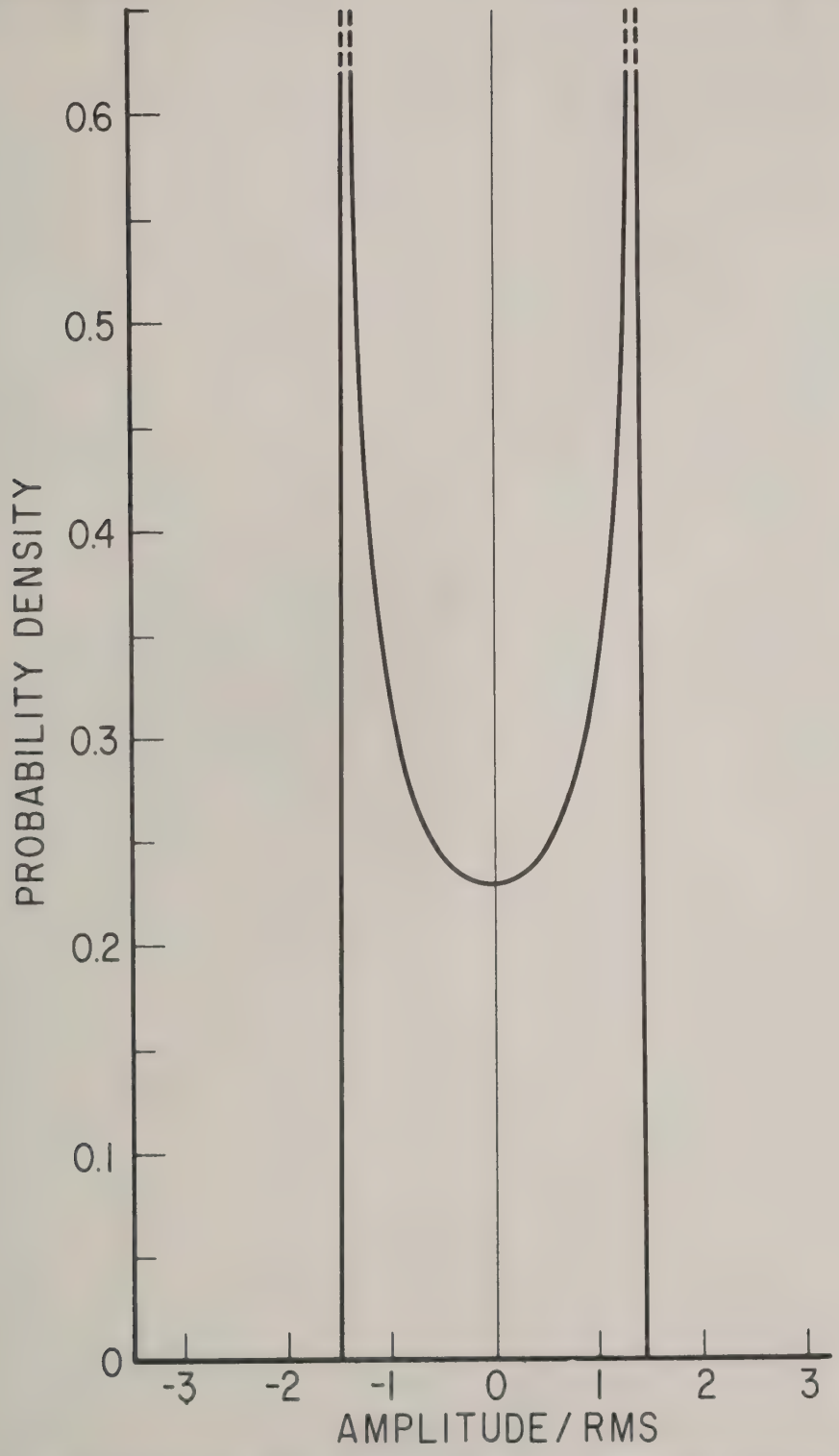


FIG. 25 PROBABILITY DENSITY OF 1 KC SINE WAVE MEASURED BY MODEL 160 B.

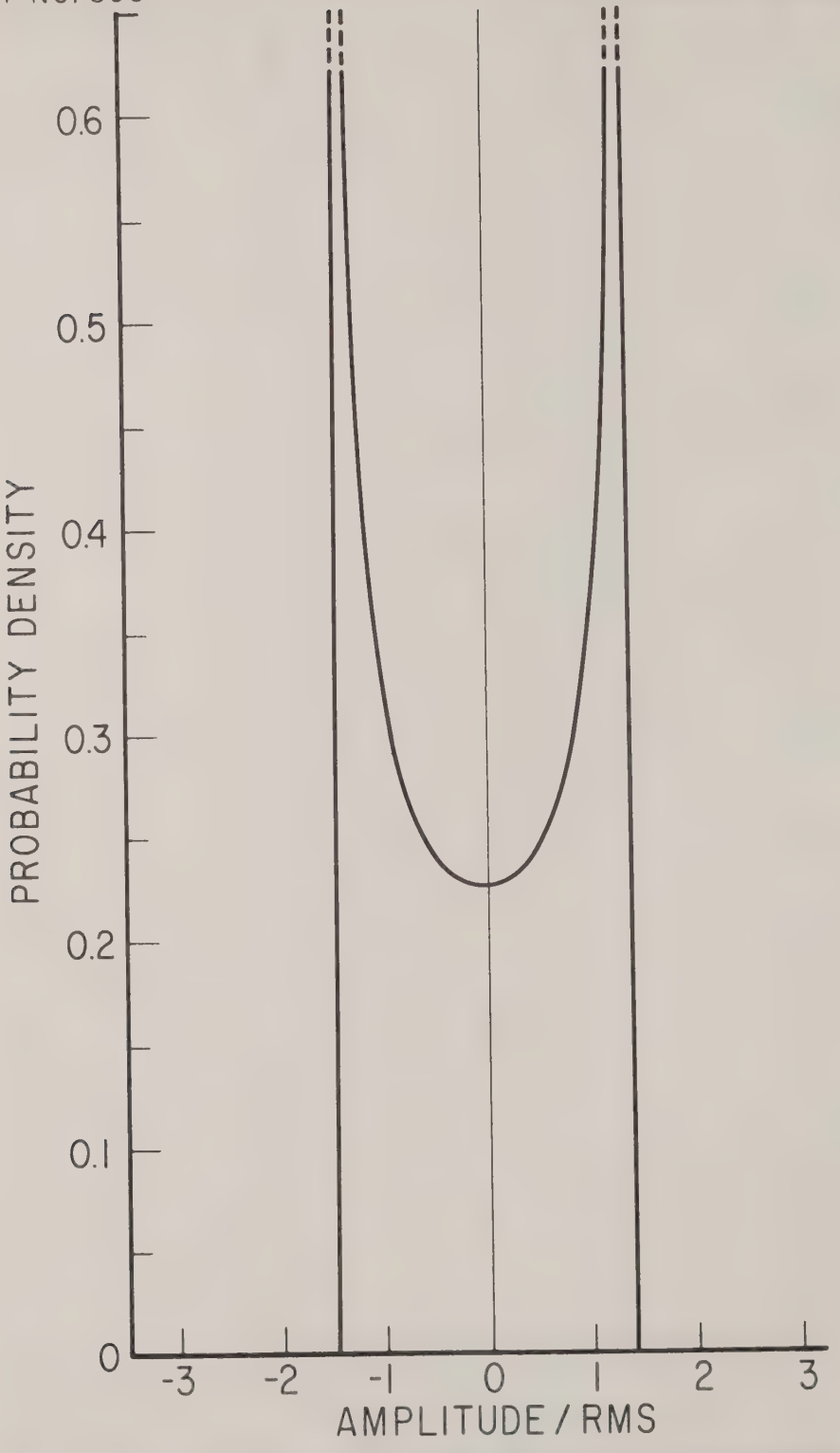


FIG. 26 PROBABILITY DENSITY OF 10 KC SINE WAVE MEASURED BY MODEL 160B

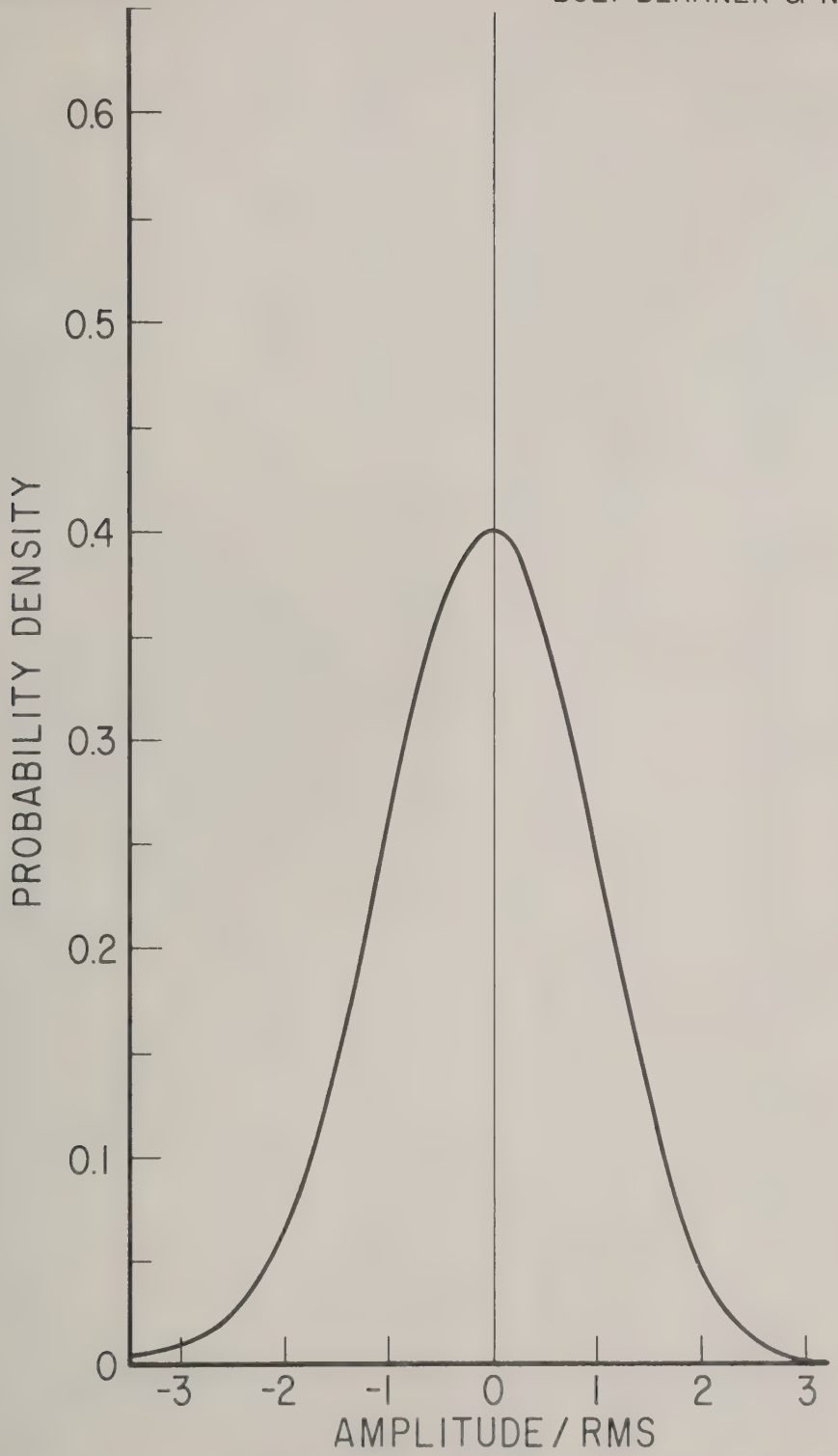


FIG. 27 PROBABILITY DENSITY OF GAS TUBE NOISE GENERATOR "SPEECH" SPECTRUM SHAPE MEASURED BY MODEL 160 B.

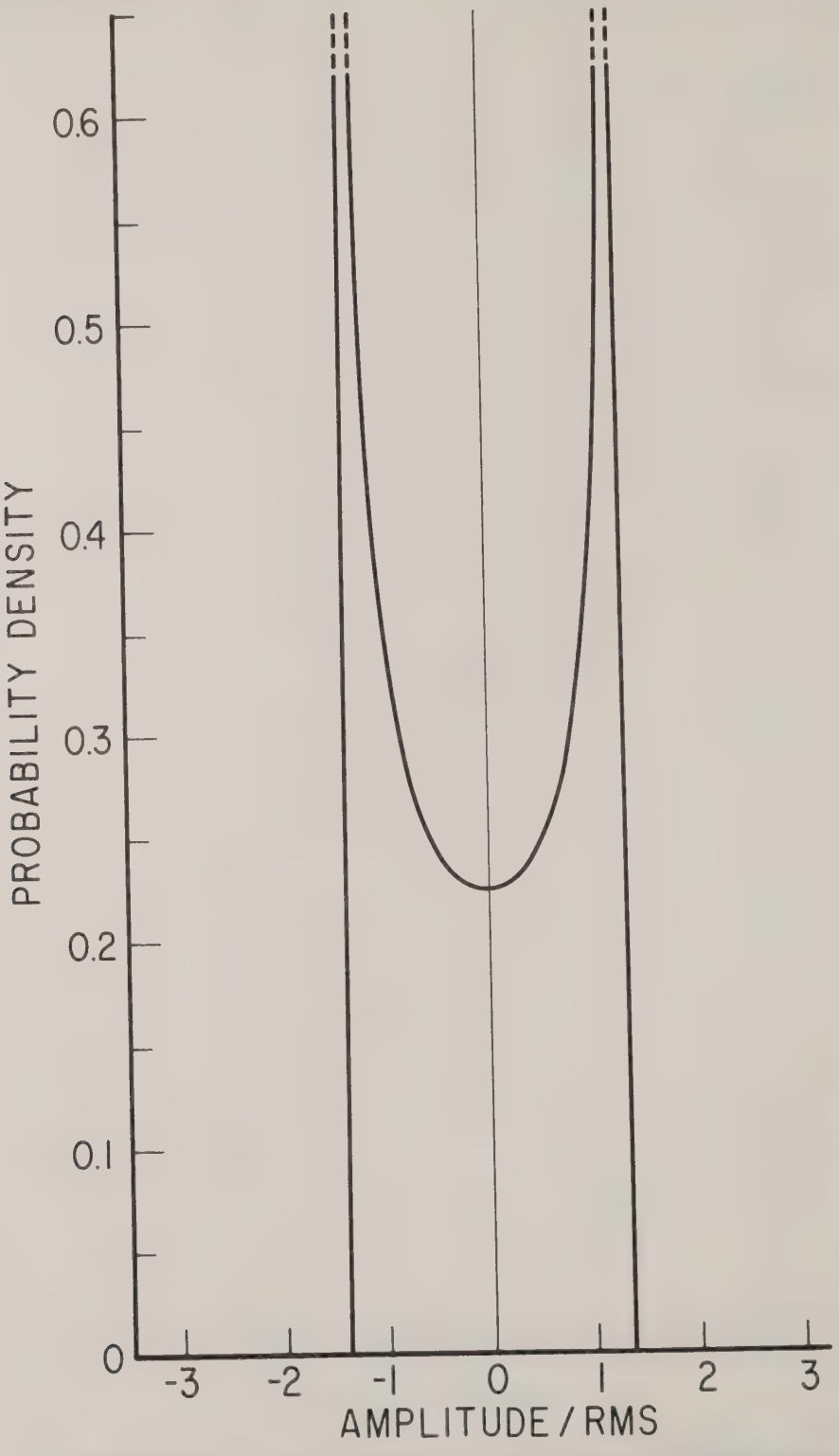


FIG. 28 PROBABILITY DENSITY OF 20 KC SINE WAVE MEASURED BY MODEL 160B

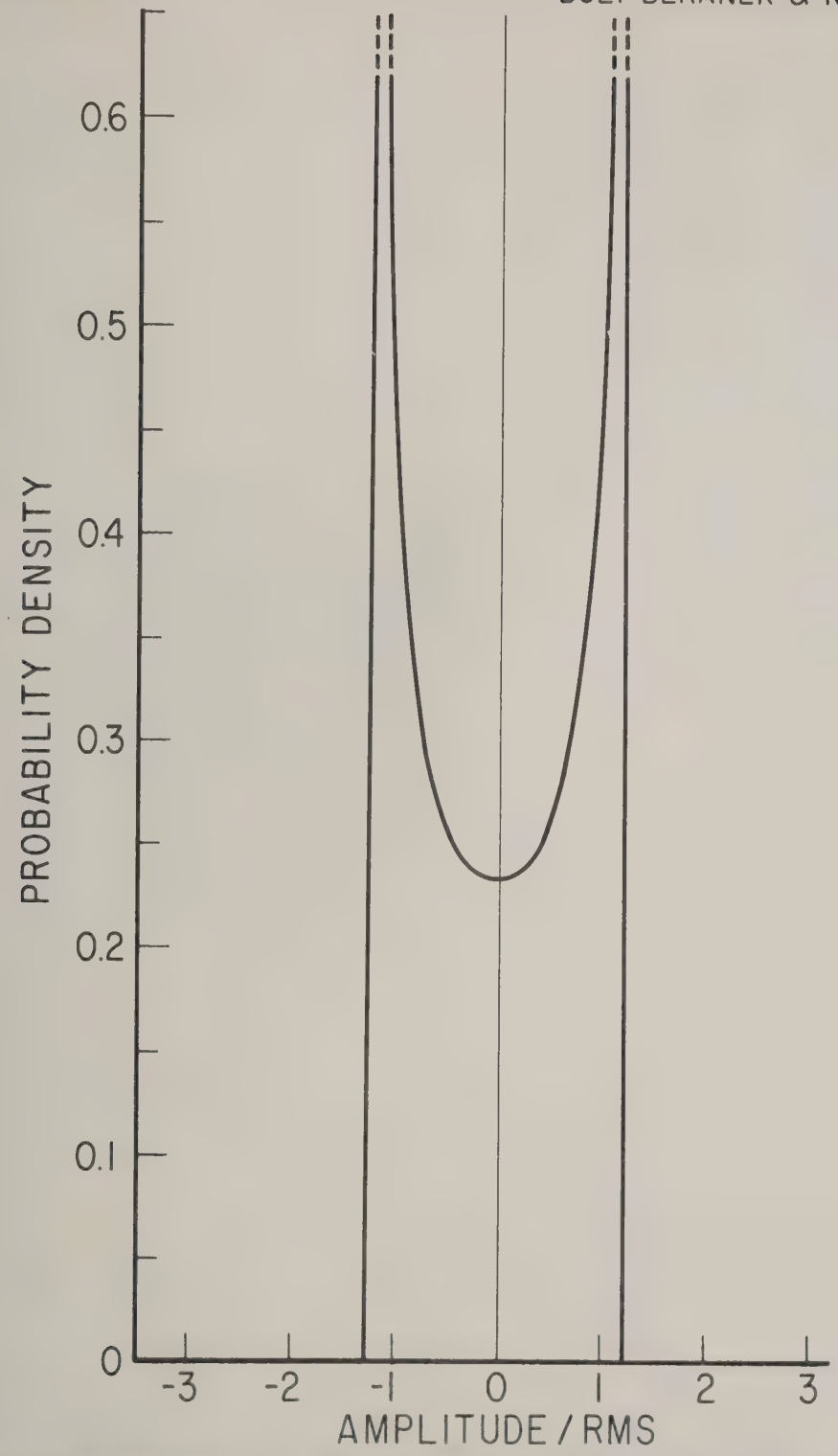


FIG. 29 PROBABILITY DENSITY OF 30 KC SINE WAVE MEASURED BY MODEL 160B

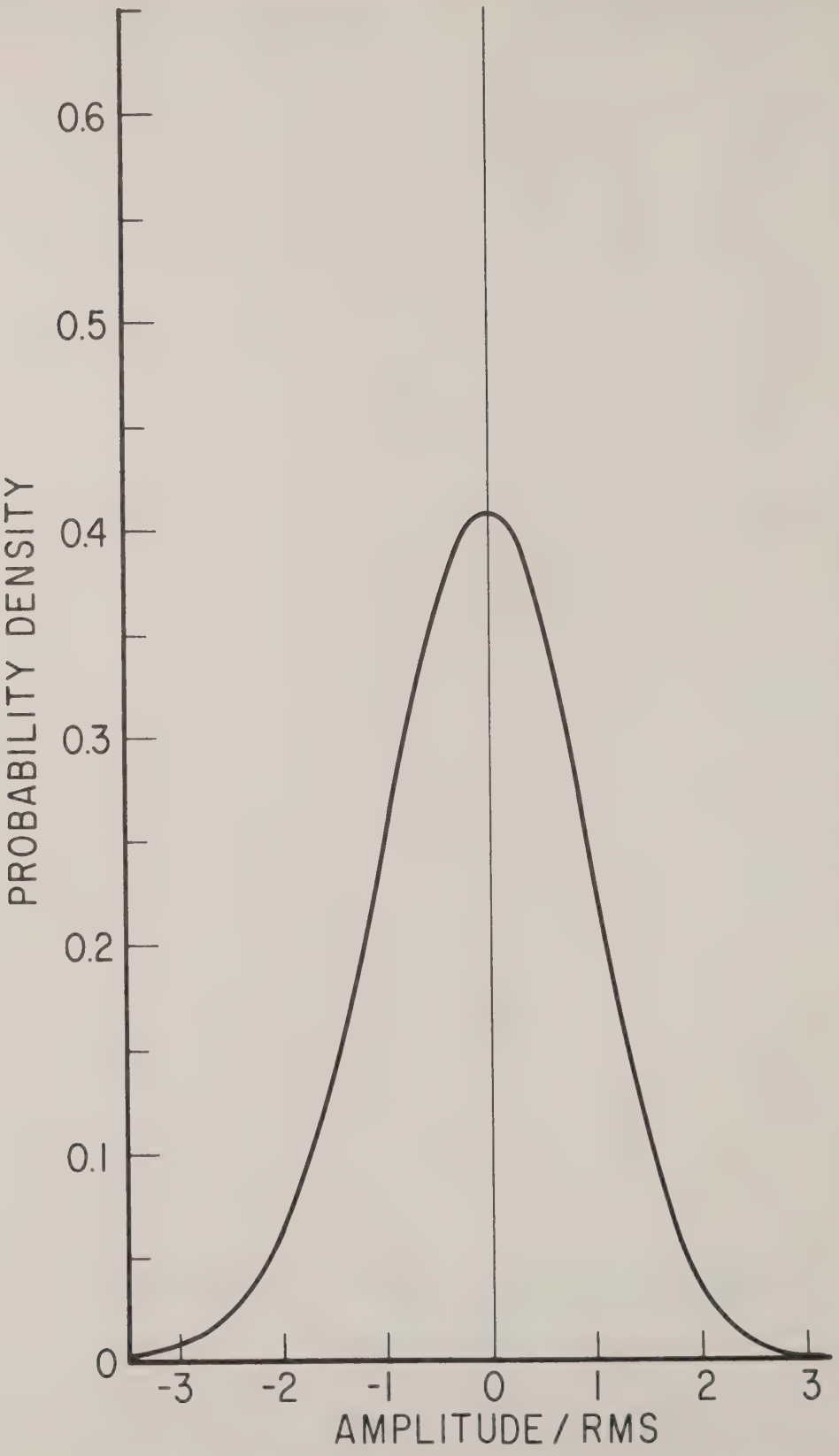


FIG. 30 PROBABILITY DENSITY OF GAS TUBE NOISE GENERATOR OUTPUT > 20 KC BANDWIDTH

is characteristic of gas tube noise generators.* In addition the probability density at the zero crossing seems to be in excess of 0.4 but has a theoretical value of .3989 for a Gaussian distribution. This is to be expected. The probability which does not occur for the large peaks (notice the steeply sloping side of the curve) must occur somewhere. This is a good example of the advantage of having the built-in normalization feature. The total integral under the curve must equal unity; the probability removed from one range of amplitude occurs in other ranges of amplitude.

Counter indications were observed for each of the measurements made with sine wave inputs. The deviations from the correct values of the probability density at the zero crossing of the sine waves as illustrated in the curves of Fig. 24 et seq. also appeared in the counter indications demonstrating some frequency dependence.

*See section V, page 34

E. Specifications

Input

Impedance:	10,000 ohms
Level:	1 volt to 50 volts RMS
Peak Factor:	±5:1
Frequency Range:	DC to 10 kc
Metering:	(1) True RMS (thermocouple meter) (2) 10 turn precision calibrated input potentiometer
Connector:	BNC coaxial
Switching:	AC/DC (AC places a lower cut off frequency of approximately 20 cycles per second)

Output

X (amplitude) axis

Impedance:	Less than 5,000 ohms (recommended load 50 k or greater)
Voltage:	-10 to +10 volts (analogous to +5 to -5 x RMS)

Connector:	BNC coaxial
Y (probability) axis analog	
Impedance:	Less than 10,000 ohms (recommended load greater than 50 k)
Voltage:	Approximately 7.5 volts for probability density of 10
Connector:	BNC coaxial
Meter Scale 1:	0 - 0.4 probability density ($\pm$ 2% full scale)
Meter Scale 2:	0 - 1 probability density ($\pm$ 2% full scale)
Digital outputs	
Impedance:	93 ohms
Voltage:	Greater than 1 volt peak
Clock frequency #1:	10 megacycles (approximate sine wave)
Clock frequency #2:	1 megacycle (approximately 10 microseconds pulses, positive and negative)
Connectors:	BNC coaxial type for 1 and 10 megacycle output
Cable:	Not more than 4 feet of RG 62u recommended when unterminated at counter

Pulse (exceedances) output

Impedance:	1,000 ohms
Rise Time:	Less than one microsecond
Amplitude:	1 volt
Occurrence:	One pulse per contribution to the probability density (with normal use, 2 pulses for exceedance of the set-in level)

Calibration

Internal:	With 60 cycle sine wave available internally
External:	Adjustments and calibration markings are available for the use of external calibrating signals

Operational Controls

Push button:	To read input r.m.s. meter
Switch:	To choose AC or DC input
"Y" 10 turn calibrated potentiometer:	Determine and set input level
Meter switch:	3 position 1 or 0.4 f.s. or "off"

Function switch:	"Balance," "Meter Zero," "Calibrate," "Use"
"X" 10 turn calibrated potentiometer:	To select amplitude at which probability is to be measured
Output-damping-sweep time control:	Select averaging time and sweep speed
Sweep switch:	Sweep forward, reverse, or off
Balance control (RC, DC and Meter Zero controls):	For balancing the bridge, input, and output respectively
Calibration controls (Cal Set and Gain):	To adjust internal calibration level and establish window width respectively
External sweep:	For applying an external voltage to determine the "X" axis voltage

Sweep Rate and Averaging Time

Switch Positions	Approximate Sweep Time From -5 to +5 r.m.s.	Averaging Time Constant
A	48 minutes	2300 milliseconds
B	24 minutes	900 milliseconds
C	12 minutes	400 milliseconds
D	6 minutes	160 milliseconds
E	3 minutes	100 milliseconds
F	1 1/2 minutes	35 milliseconds
G	No sweep	7.5 milliseconds

Accuracy

Input level: ± 0.2 db from DC to 10 kc

X (amplitude) axis: ± .05 times the r.m.s.

Probability density:

Meter output: 2% of full scale

Analog output for probability densities 10^{-3} to 10

Sine wave input: 1% of full output voltage

Noise input: 0.1% of full output voltage

Counter output for probability densities of 10^{-6} to 10: ± 5% of read value

Range

Input Characteristics:	Will give correct estimate of each contribution to the probability density from the input signal when the input signal remains in an amplitude interval of 0.1 of the r.m.s. for no less than 0.5 micro-seconds over the amplitude range from -5 to +5 times the r.m.s.
Counter output:	Probability densities from 10 to 10^{-6} can be read within the prescribed accuracy
Analog output:	From 2 to 10^{-3}
Meter:	From 1 to 5 x 10^{-2}

Power Requirements

117 volts AC, 60 cps, $\pm$ 7 1/2%

Physical Characteristics

Dimensions:	19 inch rack panel, depth: 15 inches, height: 8 3/4 inches, weight: 38 pounds

Miscellaneous

Illuminated meter as on-off indicator. All connectors are duplicated at the rear. All essential balance and calibration controls are available behind a drop-out panel at front.

APPENDIX A

UNCERTAINTY IN ESTIMATES OF THE PROBABILITY DENSITY

In section III an equation for the normalized variance of estimates of the probability density is given.

$$\sigma_n^2 = \frac{(3/8\ \pi)^{1/2}}{f_0 T\ w(x)\ \Delta x} \qquad \text{(III-4)}$$

In order to use this expression it is necessary to know what each contributing term signifies. Δx is clearly the amplitude interval of the instrument. When the PDA is used in the usual manner, that is, calibrated to obtain normalization, Δx is 0.1 x RMS. $w(x)$ is simply the measure of the probability density as obtained on the PDA.

The quantity T in the expression is not necessarily so simple. If a counter has been used to obtain the measurement of the probability density, and the counter each time totalizes over an interval T, then this is the quantity which appears in III-4. However, when other types of averaging are used, an equivalent T must be appropriately chosen. Where equivalent simply means a time constant which gives an equivalent averaging to a sampling interval of T.

For simple averaging with an RC time constant, T is to replace with 2τ where $\tau = RC$ is the time constant. For a critically damped single degree-of-freedom system such as a galvanometer T is to be replaced by 4τ. For multiple section RC low pass filters (the PDA uses 2 section RC filters) T is to be replaced by $2\Sigma\tau$ as long as the individual RC circuits do not interact. If a meter movement

is not critically damped the more exact equivalent would be to set $T = (\pi f_r Q)^{-1}$ where f_r is the resonant frequency and Q is the ratio of f_r to the frequency difference at the half power points of the meter movement frequency response. Other equivalences can be computed when required. The method is quite simple.*

f_0 in Eq. (III-4) is the equivalent cut-off frequency. In the case of a flat low pass filter with infinite attenuation outside the passband and negligible slope at the cut-off frequency, f_0 is the upper cut-off frequency. For less than ideal filters it is also a good approximation. The calculations here are quite similar to those for obtaining T. For example, for a single RC low pass filter f_0 is to be replaced by $\pi\nu_0$ where ν_0 is the frequency at the half power point of the RC filter. For a single-degree-of-freedom system with a Q sufficiently large and a center frequency ν_0, the equivalent f_0 is $(\pi\nu_0/2Q)$.

With these quantities so defined one can now compute σ_n^2. The normalized uncertainty can then be used to compute expected errors and confidence levels. Usually it is assumed that the errors are distributed normally and the σ_n^2 is the variance of the distribution of errors. To establish a confidence level one choses a set of symmetric points on the normal error curve such that the integral between the two points is equal to the desired confidence level. The distance of either of these points from the mean defines the limits of error. A single normalized variance such as σ_n^2 yields an entire table of results. To use confidence levels and percentage

*See for example the appendix of J. T. Broch and C. G. Wahrman, Effective Averaging Time of the Level Recorder Type 2305, Bruel and Kjaer Technical Review, No. 1, 1961.

error one asks the question what is the probability that the estimate will be within p per cent of the correct answer, c per cent of the time; c is the confidence interval. Tables are widely available[8] which give errors and confidence levels in dimensions of the variance.

As the uncertainty becomes large one can anticipate difficulties with such a scheme when the uncertainty refers to probability densities. If the confidence level is computed on the basis of the normal distribution it could not hold in the limit for large deviations from the probability density. The reason is simply that estimates of the probability density must be either positive or zero; the normal distribution implies negative values. The problem of determining the distribution of errors for estimates of the probability density has not yet been solved. Below are presented examples of confidence level error estimation applicable when the normalized uncertainty is reasonably small.

Example 1. Consider an input signal which has been passed through an RC low pass filter with cut-off frequency $\nu_0 = 2{,}000$ cycles. A counter is used which is set to a totalizing interval of 10 seconds. Measurements of the probability density are obtained. The probability density measured at X = 1 is 0.2 and at X = 3 is .002. What accuracy do these measurements represent with 90% confidence?

We have T = 10 seconds. For an RC low pass filter with cut-off frequency $\nu_0 = 2{,}000$ cps, $f_0 = \pi\nu_0 = 2\pi \times 10^3$. With this information we can compute σ_n^2. We obtain $\sigma_n^2 = 8.7 \times 10^{-5}$ for w(1) = 0.2 and $\sigma_n^2 = 8.7 \times 10^{-3}$ for w(3) = .002. For convenience a limited table of confidence levels is presented in Table A-I. From the table we determine that with 90% confidence one can expect the

TABLE A-I

Confidence Levels to Determine That c% of the Measurements are in Error Within $p\% = K\sigma_n\%$

c	K
98	2.33
95	1.96
93	1.81
90	1.64
88	1.55
85	1.44
82	1.34
80	1.29
78	1.23
75	1.15
73	1.10
70	1.04
68	1.--

error for the case of $w(1) = 0.2$ to be within about 1.5%. But for the lower probability density of $w(3) = .002$ it can only be expected to be within about 15%.

Example 2. It is frequently desirable to determine the required operating parameters to make a measurement which obtains an answer which has an accuracy within a given per cent at a given confidence level. The problem is to determine the averaging time required. By inverting Eq. (III-4) it can be seen that the quantity $w(x)$, the probability density to be measured is not known beforehand. How can one determine the averaging time? In such a case one would wish to use the least conservative estimate to minimize preliminary data reduction time. Such an estimate is obtained from the Bienaymé Tchebychev theorem.[16] This theorem states that the probability of the magnitude of a quantity being greater than (k) x (RMS value) is always less than or equal to $1/k^2 (k>1)$. Although no probability density uniquely corresponds to the Bienaymé Tchebychev theorem[16] one can chose a continuous function which corresponds in the worst case to a probability density (valid only for $\|x\| > 1$) of $1/2\|x^3\|$. Inserting this in Eq. (III-4) and inverting the equation we obtain the lowest possible estimate for the required time interval to obtain a measurement of probability density at X of any function with "noise bandwidth" f_0.

$$T = \frac{3.44\ \|x^3\|}{f_0 \sigma_n^2} \qquad \text{(A-1)}$$

where σ_n^2 is to be determined from a table such as Table A-I.

Consider as an example that the signal has been passed through a 1/3 octave filter centered at 1,000 cps and an accuracy of 10% at 80% confidence is desired. For f_0 one can use $f_0 = 1.57 \times .26 \times 10^3 = 408$ cps equivalent bandwidth if a singly resonant filter is used or one could use 260 cps if an ideal filter is assumed. For this example we chose 300 cps as a compromise figure. From the Table A-1 it can be seen that 10% accuracy with 80% confidence requires $\sigma_n^2 = 10^{-2}/(1.3)^2 \simeq 5.92 \times 10^{-3}$. Then an estimate of the probability density at X = 2, for example requires an averaging time of at least 16.5 seconds if the accuracy is to be specified as being in error no more than 10% for 80% of a set of repeated measurements.

Note that after obtaining an estimate of the probability density the uncertainty can be calculated. The above minimum averaging time merely establishes the lower limit of averaging time. For Gaussian distributed signals for example, Eq. (A-1) must be corrected by the factor

$$\frac{\sqrt{2\pi}\, e^{+x^2/2}}{\|x^3\|} \qquad \text{(A-2)}$$

From this example it can be seen that a uniform sweep time as available on the PDA makes the uncertainties such as to give a much better estimate of the probability density for small values of x than it does for large values of x. This is of course entirely consistent with the usual requirements of sound and vibration. The available sample durations are seldom long enough to obtain a really good estimate of the probability density for large values of x. Furthermore when precise measurements are

required, a counter is required. The use of the internal sweep is no longer feasible.

Example 3. In this example we present an operational method by which the uncertainty can itself be used to determine the sampling time required for each value of the probability density. In the derivation of Eq. (III-4) the frequency dependence was introduced as a sort of root mean square frequency. (This is true only of the Fox derivation[6,7].) The argument contends that the larger the number of contributions to the probability density, that is, the larger the number of times the signal goes into the amplitude inverval, Δx the less the uncertainty. Using this principle one can utilize the PULSE-OUT of the PDA to time the sampling interval.

The PULSE-OUT operates a totalizer. The totalizer is pre-set at some value to be determined subsequently. Each time it reaches this value it terminates a sampling interval. A new sampling interval is then begun to measure the next value of the probability density. It can be seen qualitatively that as the probability density becomes small the number of pulses per unit time from the PULSE-OUT will become less and it will therefore take a longer time for the totalizer to accumulate a specified count. In this way longer sampling intervals are obtained for lower probability densities. Quantitatively the expression for the total number of counts required to achieve a specific uncertainty is very simply

$$N = \frac{1}{\sigma_n^2 \, \Delta x} \qquad \text{(A-3)}$$

This expression contains no reference to the frequency nor to the probability density since both these quantities determine the number of counts that are achieved. It is therefore a beautifully simple way of achieving a specified uncertainty.

If all estimates were required to be within 10% with 80% confidence then by applying Eq. (A-3) one immediately obtains that it is necessary to register a total of 1.69×10^3 counts at the PULSE-OUT jack for each measurement of the probability density, irrespective of input frequency and value of the probability density.

APPENDIX B

EXAMPLES OF COMMON PROBABILITY DENSITIES

For testing and comparison purposes it is desirable to have some knowledge as to what probability densities are expected for certain well known waveforms. Several of these probability densities have been illustrated in the text. In this section we merely list the formulae for the probability densities of several well known waveforms. To demonstrate the manner in which such formulae are derived, the probability density of a sine wave is derived by the method of characteristic functions.

In the method of characteristic functions one obtains the average of the quantity exp iρy, where y = f(t) is the function for which one wishes to obtain the probability density. The Fourier transform of this average is then obtained from the ρ domain to the y domain. The result is the probability density of y. The basis for this technique is in the literature.[16] We apply this method to a sine wave.

$$\langle e^{i\rho y} \rangle = \frac{1}{T}\int_0^T e^{i\rho A \sin\omega t}\, dt \qquad \text{(B-1)}$$

$$= J_0(A\rho)$$

Where J_0 indicates the zero order Bessel function. The Fourier transform of Eq. (B-1) yields the probability density.

$$w(y) = \int J_0(A\rho)e^{-i\rho y}d\rho = \frac{1}{\pi\sqrt{A^2 - y^2}} \qquad |y| \leq A$$

$$= 0 \qquad |y| > A \tag{B-2}$$

Although in this case the probability density could have been obtained by more direct methods, one should recognize that the method of characteristic functions is a very powerful tool for more complicated functions.

Other probability densities are listed below. For a square wave one obtains

$$w(y) = \frac{1}{2}\left[\delta(y-A) + \delta(y+A)\right]$$

where

$$A = \text{peak amplitude} \tag{B-3}$$

and

$$\delta(y) = \text{Dirac delta function.}$$

For a triangular wave symmetric about the zero axis one obtains

$$w(y) = \frac{1}{2A} \qquad |y| \leq A$$

$$w(y) = 0 \qquad |y| > A \quad . \tag{B-4}$$

For the sum of a square wave and independent sine wave one obtains for $|y| \leq (A + B)$

$$w(y) = \frac{1}{2\pi} \left| \frac{(A^2 - B^2 - y^2 - 2By)^{1/2} + (A^2 - B^2 - y^2 + 2By)^{1/2}}{[(A^2 - B^2 - y^2)^2 - 4B^2y^2]^{1/2}} \right|$$

where

(B-5)

A = peak amplitude of sine wave

and

B = peak amplitude of square wave.

The normal distribution is

$$w(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

where

(B-6)

$\mu = \langle y \rangle$

and

$$\sigma^2 = \langle (y-\mu)^2 \rangle$$

The envelope of narrow band noise follows the Rayleigh distribution

$$w(y) = \frac{y}{\sigma^2} e^{-\frac{y^2}{2\sigma^2}} \qquad y \geq 0 \qquad \text{(B-7)}$$

The above limited list of commonly encountered distributions should facilitate identifying measurements obtained with the PDA.

APPENDIX C

DEVIATION COEFFICIENTS

When a density function deviates only slightly from another density function which has a closed analytic form it is often convenient to specify parameters which expressly describe the deviation rather than to plot the curve in its entirety. Such deviation parameters exist because of the possibility of expanding a general probability function in a series which converges in the first order to a well known density function.

The best known example of such a series is the Edgeworth series which converges to the Gaussian function. The form of this series is as follows:

$$w(x) = \frac{1}{\sqrt{2\pi}} \; e^{-\frac{x^2}{2}} \left(1 + \sum_{i=3}^{\infty} \frac{C_i}{i!} \; H_i(x) \right)$$

Where

$$x = \frac{y}{\sigma}$$

$H_i(x)$ = the ith Hermite Polynomial[17]

And

$$C_3 = \langle x^3 \rangle$$

$$C_4 = \langle x^4 \rangle - 3$$

$$C_5 = \langle x^5 \rangle - 10\langle x^3 \rangle$$

$$C_6 = \langle x^6 \rangle - 15\langle x^4 \rangle + 30$$

etc.

Consider the first two coefficients of this expansion. The coefficient of Skewness is defined as the ratio of third moment to the cube of the standard deviation. The coefficient of Kurtosis is defined as the ratio of the fourth moment to the fourth power of the standard deviation. One can then specify values of these parameters such that at least the first two correction terms of the series will vanish. That is, if the coefficient of Skewness is equal to zero and the coefficient of Kurtosis is equal to 3 and the probability density otherwise looks Gaussian then it is highly probable that it is Gaussian. A convenient method for determining the coefficients of Skewness and Kurtosis from the probability density curve is to plot normalized overlays of probability functions given by the first few terms of the Edgeworth series with specific values of the coefficient of Skewness and Kurtosis. These then are compared with the curves plotted with the PDA and information about the Skewness and Kurtosis extrapolated.

The coefficient of Skewness is as its name implies a measure of the Skewness of the probability density. In Figures 13 and 14 in this report, the probability densities of the noise from a gas tube and solid state noise generator exhibit coefficients of Skewness which are not zero. Such asymmetry however can often be observed on an oscilloscope. Observation of a gas tube noise generator using a comparatively slow sweep so that the density of

the trace on the oscilloscope gives some measure of probability shows up quite distinctly that the probability density is asymmetric.

The coefficient of Kurtosis is a little less accessible. It measures the distortion of the tails of the distribution.

Deviation coefficients beyond those for Skewness and Kurtosis are implicit in expansions such as the Edgeworth expansion. One might consider for example the ratio of the fifth moment to the fifth power of the standard deviation. Generally such coefficients are not used however. The reason is simply if the curve deviates considerably from a Gaussian curve then the Edgeworth expansion is not satisfactory. However, when a highly precise estimate of the symmetry of a probability density function is desired the higher order odd moments serve as very good indicators.

APPENDIX D

METHODS OF CALIBRATION

Three types of calibration for the PDA are available. The first type is discussed in the text. It uses an internal calibration signal expressly designed for the purpose of calibration. A second type of calibration uses derived measurements which determine such quantities as the normalization factors. The third category of calibration techniques involve the use of standardized waveforms in the actual plotting and measuring of probability density.

Internal calibration of the PDA consists of an adjustment of a gain control. The sole function of this gain control is to establish a proper amplitude interval. The remaining circuits of the PDA have been established such that with the proper amplitude interval (0.1 x RMS) the output is normalized.

A simple direct calibration of the amplitude interval can be performed as follows. The input is short circuited. The meter scale switch is placed in the 1.0 position. The "X" potentiometer is rotated near the zero point observing the probability density meter. When the "X" potentiometer is close to zero the probability density meter will read off scale. When the "X" potentiometer is removed from zero the probability density meter will have no reading. The potentiometer is rotated back and forth locating those points which correspond to the transition from zero to an off-scale reading. The difference between these points read on the scale of the "X" dial is the amplitude interval. If a counter is available it may be used instead of the probability density meter to determine at which point the gate opens. The PULSE-OUT may be used or either the 1 or 10 mc outputs.

Calibration with a known waveform is obtained by using equations such as those given in Appendix B. One carefully computes the probability then applies the signal to the PDA and adjusts the analyzer to obtain the correct reading. The PDA has no internal circuitry which can discrininate between the accuracy obtained at the zero crossing or at higher amplitudes. In general therefore calibration just at zero amplitude is adequate. If calibration is desired over the full range, then a suitable signal to use would be a triangular wave, since it should have equal probability over the entire range.

Sine waves are more readily available and a simple calculation with a sine wave equation shows that a sine wave of 1 volt RMS should have a probability density at the amplitude zero of approximately 0.225. Measuring the probability density of sine waves at the zero crossing, maintaining the amplitude and varying the frequency is a means of obtaining a measure of the instrument's frequency response. The use of sine waves for obtaining the calibration of the instrument at some amplitude other than the zero crossing is not advised, since the slope of the probability density is too steep. Correction series such as Equations (5) of section III when applied to such a steep probability density indicate that large local errors can occur.

It is important to emphasize that random waveforms are not recommended as means of calibration. The uncertainty relationships as discussed in Appendix A and the text indicate that extremely long time averages would have to be used to obtain accurate enough estimates of probability density to be useful in calibration. Furthermore, it is questionable whether there are commercially available random noise sources which have true Gaussian distributions. In general, it is therefore desirable to calibrate the PDA with deterministic waveforms.

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INDEX

Amplifier, thermocouple, 64
Amplitude identification output, 15, 16
Amplitude interval, 6, 12, 14, 17, 69
    calibration, 40
    circuit, 69
    relation to time interval, 19
Amplitude level, 14
Applications, 23 et seq, 34
Arguimbau circuit, 2, 20, 68
Asymmetry, 34, 35
Averaging, 15
Averaging time constants, 96
Axis shift, 17

Balance circuits, 71
Balance controls, adjustment, 40
Bandwidth requirements, 43

Calibration of PDA, General, 39
    internal circuit, 66
    of PDA with X-Y Recorder, 43
    to obtain amplitude density, 60
    to obtain distribution of peaks, 54, 56
    to obtain unnormalized exceedance, 63
    to use PULSE-OUT, 54
    with counter, 48
    with Graphic Level Recorder, 52
    with oscilloscope, 46
Central limit theorem, 12, 28
Chart speed for 2305, 52
Coincidence, 59
Conditional probability, 34, 59
Confidence intervals, 10
Confidence levels, A-2
Counter, significance of readings, 49, 50
Counter, threshold adjustment, 48

Data processing, 34
"DC pulse limit", 70, 77, 80
Diode noise, 34, 36
Distribution of peaks, 54

INDEX CONT.

Electroencephelographs, 29
Ensemble statistics, 28, 30, 31, 32, 34, 59
Envelope method, 56
Error, 12
Estimator, 8, 11
Exceedances, 53, 54
to control uncertainty, A-7

Fatigue, 25
Fluctuations, 29
on counter, 51
on PDA meter, 43

Gas tube, 34, 35
Gates, output, 80
Gaussian distribution, 12

Human operator characteristics, 29
Hydrodynamic fluctuations, 29

I. F. circuits, 67
alignment, 74
Input level, calibration, 41
Input meter protection, 42
Interacting modes, 34

Joint probability, 34, 58

Limiter, cathode coupled, 69, 70
Logarithmic probability, 52

Meter shunt, 71
Micrometerological fluctuations, 29
Modulator, 66

Noise bandwidth, 10
equivalents, A-1, A-2
Noise generators, 34
Non-linear, 25, 26
Normalization, 8, 9, 14, 20, 55, 60

Operational procedures, 37
for amplitude density, 59
for joint probability density, 58

INDEX CONT.

for obtaining distribution of energy, 58
for obtaining distribution of functions of the input, 57
for obtaining distribution of peaks, 54, 56
for obtaining exceedances, 53
for obtaining unnormalized exceedances, 63
with counter, 48
with external sweep, 46
with graphic level recorder, 32
with X-Y recorder, 43
Oscillator alignment, 74, 80
Oscilloscope readout, 24
Output damping, circuit theory, 70
Output zero, 40
malfunction, 80
Overshoot, 77

Powell Method, 54
Power supply, 72, 73
Probability, definition, 5, et seq.
Probability density, definition, 5 et seq.
Propagation, 29, 33
PULSE-OUTPUT, 53

Random waveforms, 5
Rate meter, 56
Reference RMS, 17
Reverberant field, 29, 33
Room statistics, 29, 33

Schmidt trigger, 70
"Set-in" level, 14, 17
Signal processing, 17, 18
Simulated noise, 25, 27
Speech, 29
Stability, 39, 53
Standard deviation, 8, 9
Structure studies, 25
Sweep rates, 96

τ_i, Time intervals, 11, 15
relationship to Δx, 19
Telephone signals, 25, 28
Temperature fluctuations, 29
Time interval, equivalents, A-1, A-2

INDEX CONT.

Time resolution, 13
Time statistics, 34, 59
Transients, 59, 62
"Trigger level adjust", 70, 74
"Trigger limit", 69, 74
Tuned circuit response, 25, 27

Uncertainty, 10, A-I
Use of PDA, general, 42
for obtaining distribution of peaks, 55, 57
to obtain amplitude density, 61
to obtain exceedances, 53
to obtain unnormalized exceedances, 63
with counter, 50
with graphic level recorder, 53
with oscilloscope, 47
with X-Y recorder, 45

Variance, 10

Window width, 12

Zero crossings, 53, 55